Met Office

Rapid update cycling with delayed observations

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Rapid update cycling - practical and theoretical considerations

Practical: benefits, methods to use, issues

Theoretical: A basic question: What is the best analysis/forecast at any instant in time?

At first sight one might suppose that this is addressed by classical estimation theory (Bayes, Kolmogorov, in the linear Gaussian case Kalman, etc), but what happens if (as is true in weather forecasting) the obs are not received instantaneously?

Define *rapid update* cycling as any cycling where observations are assimilated as soon as they are received, or within some short time of receipt, eg within one hour

NB - Term 'RUC' has been used elsewhere to denote a specific operational system (NCEP)

[Ref] 'Rapid update cycling with delayed observations'
Tellus A: Dynamic Meteorology and Oceanography
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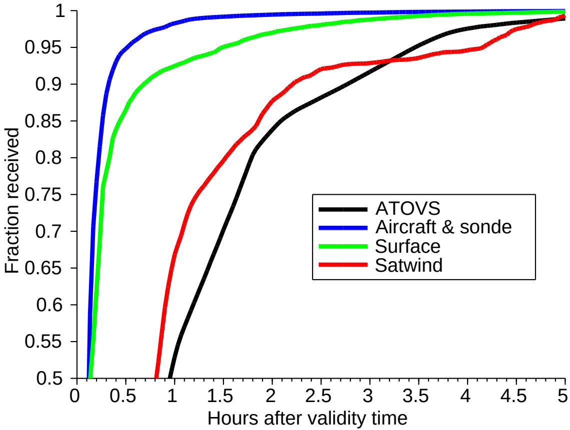
Delays in receiving obs



Delay in receiving various observation types in the Met Office observation processing system, for observations valid between 9Z on 15 June 2015 and 3Z on 18 June 2015.

Why bother doing rapid update cycling?

- Advantages stemming from timely use of obs:
  - At any instant in time have global forecast using latest obs
  - Improve lateral boundary conditions for limited area models
- Advantages stemming from higher frequency of cycling:
  - analysis never departs far from background
    $\rightarrow$ smaller increment
    $\rightarrow$ reduced impact of nonlinearity (cf outer loop)
  - assimilation cost spread over time (parallelism in time)

Notation for observation validity and availability times

As an example
(a) suppose all obs are received within 3 hours
(b) we batch obs to nearest hour

Eg, at 4Z receive obs valid at 1Z,2Z,3Z and 4Z

		Observation validity time					
		$1Z$	$2Z$	$3Z$	$4Z$	$5Z$	$6Z$
Obs	$4Z$	$\mathbf{y}_1^{(4)}$	$\mathbf{y}_2^{(4)}$	$\mathbf{y}_3^{(4)}$	$\mathbf{y}_4^{(4)}$		
receipt	$5Z$		$\mathbf{y}_2^{(5)}$	$\mathbf{y}_3^{(5)}$	$\mathbf{y}_4^{(5)}$	$\mathbf{y}_5^{(5)}$	
time	$6Z$			$\mathbf{y}_3^{(6)}$	$\mathbf{y}_4^{(6)}$	$\mathbf{y}_5^{(6)}$	$\mathbf{y}_6^{(6)}$

Table: Notation for observation validity and availability times, here $N = 3$

Some Features of optimal solution in linear Gaussian case

Example: how to update from window $\{1,2,3,4\}$ at $t=4$ to window $\{2,3,4,5\}$ at $t=5$:

From $t=4$ have analyses $\mathbf{x}_{a,1}^{(4)}, \mathbf{x}_{a,2}^{(4)}, \mathbf{x}_{a,3}^{(4)}, \mathbf{x}_{a,4}^{(4)}$

and $4n \times 4n$ analysis error covariance matrix

$$\begin{pmatrix} A_{11}^{(4)} & A_{12}^{(4)} & A_{13}^{(4)} & A_{14}^{(4)} \\ A_{21}^{(4)} & A_{22}^{(4)} & A_{23}^{(4)} & A_{24}^{(4)} \\ A_{31}^{(4)} & A_{32}^{(4)} & A_{33}^{(4)} & A_{34}^{(4)} \\ A_{41}^{(4)} & A_{42}^{(4)} & A_{43}^{(4)} & A_{44}^{(4)} \end{pmatrix}$$

At $t=5$ have just received obs $\mathbf{y}_2^{(5)}, \mathbf{y}_3^{(5)}, \mathbf{y}_4^{(5)}, \mathbf{y}_5^{(5)}$.

Our prior estimate of states $\mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4, \mathbf{x}_5$ at $t=5$ is

$$\underline{\mathbf{x}}_b^{(5)} \equiv \begin{pmatrix} \mathbf{x}_{b,2}^{(5)} \\ \mathbf{x}_{b,3}^{(5)} \\ \mathbf{x}_{b,4}^{(5)} \\ \mathbf{x}_{b,5}^{(5)} \end{pmatrix} = \begin{pmatrix} \mathbf{x}_{a,2}^{(4)} \\ \mathbf{x}_{a,3}^{(4)} \\ \mathbf{x}_{a,4}^{(4)} \\ \mathcal{M}_4^5 \mathbf{x}_{a,4}^{(4)} \end{pmatrix}$$

Some Features of optimal solution in linear Gaussian case, cont'd

Then posterior estimate of states $\mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4, \mathbf{x}_5$ at $t = 5$ is

$$\underline{\mathbf{x}}_a^{(5)} \equiv \begin{pmatrix} \mathbf{x}_{a,2}^{(5)} \\ \mathbf{x}_{a,3}^{(5)} \\ \mathbf{x}_{a,4}^{(5)} \\ \mathbf{x}_{a,5}^{(5)} \end{pmatrix} = \underline{\mathbf{x}}_b^{(5)} + \underline{\boldsymbol{\delta}} = \begin{pmatrix} \mathbf{x}_{a,2}^{(4)} \\ \mathbf{x}_{a,3}^{(4)} \\ \mathbf{x}_{a,4}^{(4)} \\ \mathcal{M}_4^5 \mathbf{x}_{a,4}^{(4)} \end{pmatrix} + \underline{\boldsymbol{\delta}}$$

where

$$\underline{\boldsymbol{\delta}} = (\boldsymbol{\delta}_2^T, \boldsymbol{\delta}_3^T, \boldsymbol{\delta}_4^T, \boldsymbol{\delta}_5^T)^T$$

minimises

$$J_b(\underline{\boldsymbol{\delta}}) + J_o(\underline{\boldsymbol{\delta}}) + J_q(\underline{\boldsymbol{\delta}})$$

where $J_o(\underline{\boldsymbol{\delta}})$ has the usual form

$$J_o(\underline{\boldsymbol{\delta}}) = \frac{1}{2} \sum_{j=2}^{5} \left[\mathbf{y}_j^{(5)} - \left(\mathbf{x}_{b,j}^{(5)} + \boldsymbol{\delta}_j \right) \right]^T R_j^{-1} \left[\mathbf{y}_j^{(5)} - \left(\mathbf{x}_{b,j}^{(5)} + \boldsymbol{\delta}_j \right) \right]$$

Some Features of optimal solution in linear Gaussian case, cont'd

and

$$J_q(\underline{\boldsymbol{\delta}}) = \frac{1}{2}\left(\boldsymbol{\delta}_5 - M_4^5\boldsymbol{\delta}_4\right)^T \left(Q_4^5\right)^{-1}\left(\boldsymbol{\delta}_5 - M_4^5\boldsymbol{\delta}_4\right)$$

but now have

$$J_b(\underline{\boldsymbol{\delta}}) = \frac{1}{2}\begin{pmatrix} \boldsymbol{\delta}_2 \\ \boldsymbol{\delta}_3 \\ \boldsymbol{\delta}_4 \end{pmatrix}^T \begin{pmatrix} A_{22}^{(4)} & A_{23}^{(4)} & A_{24}^{(4)} \\ A_{32}^{(4)} & A_{33}^{(4)} & A_{34}^{(4)} \\ A_{42}^{(4)} & A_{43}^{(4)} & A_{44}^{(4)} \end{pmatrix}^{-1} \begin{pmatrix} \boldsymbol{\delta}_2 \\ \boldsymbol{\delta}_3 \\ \boldsymbol{\delta}_4 \end{pmatrix}$$

where 'big B'

$$\begin{pmatrix} A_{22}^{(4)} & A_{23}^{(4)} & A_{24}^{(4)} \\ A_{32}^{(4)} & A_{33}^{(4)} & A_{34}^{(4)} \\ A_{42}^{(4)} & A_{43}^{(4)} & A_{44}^{(4)} \end{pmatrix}$$

is obtained from analysis error covariance at previous stage by shearing off oldest row and column.

Approximation

So optimal solution has

$$J_b(\underline{\boldsymbol{\delta}}) + J_q(\underline{\boldsymbol{\delta}}) =$$

$$\frac{1}{2}\begin{pmatrix} \boldsymbol{\delta}_2 \\ \boldsymbol{\delta}_3 \\ \boldsymbol{\delta}_4 \end{pmatrix}^T \begin{pmatrix} A_{22}^{(4)} & A_{23}^{(4)} & A_{24}^{(4)} \\ A_{32}^{(4)} & A_{33}^{(4)} & A_{34}^{(4)} \\ A_{42}^{(4)} & A_{43}^{(4)} & A_{44}^{(4)} \end{pmatrix}^{-1} \begin{pmatrix} \boldsymbol{\delta}_2 \\ \boldsymbol{\delta}_3 \\ \boldsymbol{\delta}_4 \end{pmatrix} +$$

$$\frac{1}{2}\left(\boldsymbol{\delta}_5 - M_4^5\boldsymbol{\delta}_4\right)^T \left(Q_4^5\right)^{-1} \left(\boldsymbol{\delta}_5 - M_4^5\boldsymbol{\delta}_4\right)$$

What happens if we replace this by

$$J_b(\underline{\boldsymbol{\delta}}) + J_q(\underline{\boldsymbol{\delta}}) =$$

$$\frac{1}{2}\boldsymbol{\delta}_2^T \left(A_{22}^{(4)}\right)^{-1} \boldsymbol{\delta}_2 + \frac{1}{2}\sum_{j=2}^{4} \left(\boldsymbol{\delta}_{j+1} - M_j^{j+1}\boldsymbol{\delta}_j\right)^T \left(Q_j^{j+1}\right)^{-1} \left(\boldsymbol{\delta}_{j+1} - M_j^{j+1}\boldsymbol{\delta}_j\right)$$

Simplification of optimal method if $Q = 0$

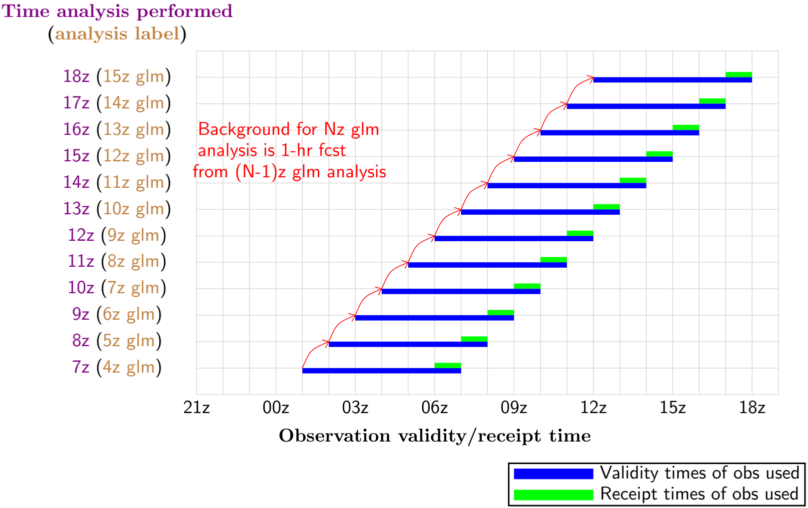
It turns out (Ref, §5) the error in this approximation depends entirely on Q. In the absence of model error, then at every stage, the $(N+1)n \times (N+1)n$ posterior error covariance matrix factors:

$$\begin{pmatrix} A^{(j)}_{j-3,j-3} & A^{(j)}_{j-3,j-2} & A^{(j)}_{j-3,j-1} & A^{(j)}_{j-3,j} \\ A^{(j)}_{j-2.j-3} & A^{(j)}_{j-2,j-2} & A^{(j)}_{j-2,j-1} & A^{(j)}_{j-2,j} \\ A^{(j)}_{j-1,j-3} & A^{(j)}_{j-1,j-2} & A^{(j)}_{j-1,j-1} & A^{(j)}_{j-1,j} \\ A^{(j)}_{j,j-3} & A^{(j)}_{j,j-2} & A^{(j)}_{j,j-1} & A^{(j)}_{j,j} \end{pmatrix} =$$

$$\begin{pmatrix} I \\ M^{j-2}_{j-3} \\ M^{j-1}_{j-3} \\ M^{j}_{j-3} \end{pmatrix} A^{(j)}_{j-3,j-3} \begin{pmatrix} I & (M^{j-2}_{j-3})^T & (M^{j-1}_{j-3})^T & (M^{j}_{j-3})^T \end{pmatrix}$$

So if $Q = 0$ only need to carry and manipulate $n \times n$ error covariance matrices.

One way of doing rapid update cycling: sliding/overlapping windows



Why not simply do sliding/overlapping windows?

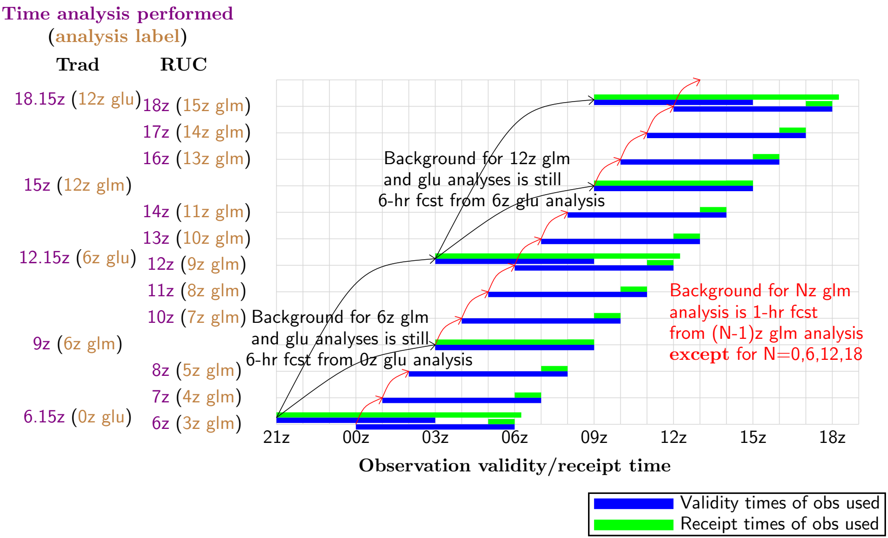
Only assimilate about $\frac{1}{6}$ as many obs per cycle as usual, which wouldn't matter if we had a fully cycled B, but the use of a fixed B favours assimilating as many obs as possible simultaneously

Suppose at every time level have two obs which are to be assimilated using 3D-Var, either simultaneously with fixed B, or one after the other, the first with fixed B_1 and the second with fixed B_2

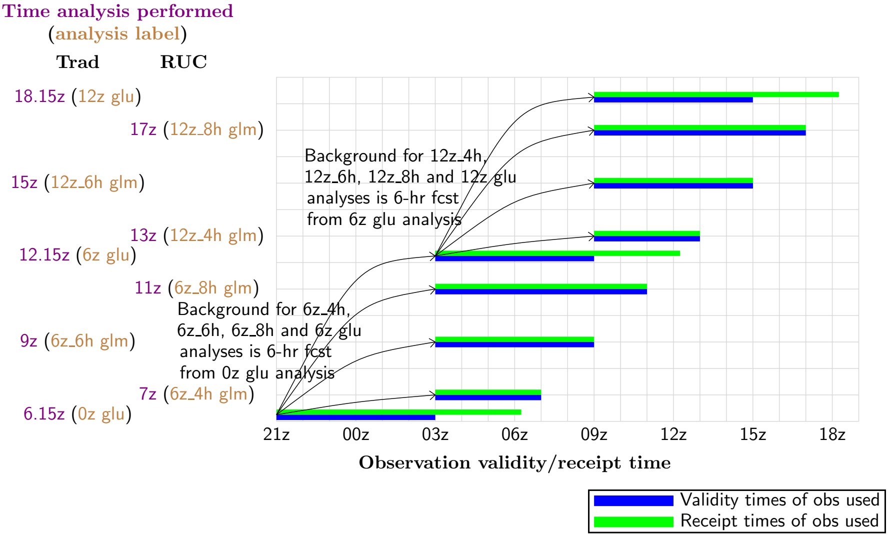
[Ref] shows that no matter how well B_1 and B_2 are chosen, we can choose B so that the mean error in the simultaneous method is lower than that in the sequential method

NB - Not same as long window argument (covariance evolution)

Method 1: Hybrid of conventional cycling and sliding windows



Method 2: expanding windows



Advantages stemming from timely use of obs (global)

- Advantages stemming from timely use of obs:
  - At any instant in time have global forecast using latest obs

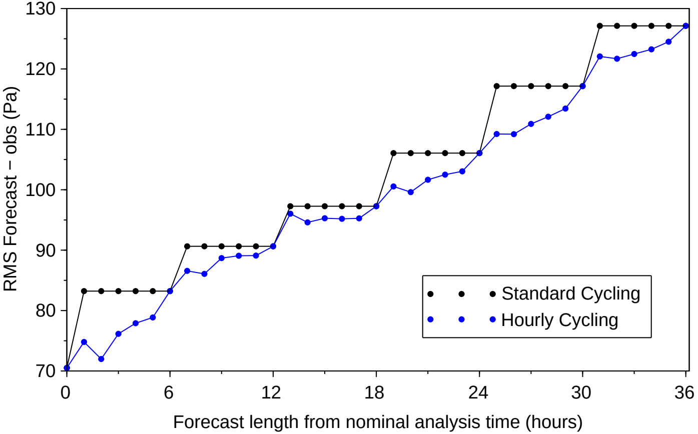


Figure: RMS error (forecast - obs) for northern hemisphere (30-90N) pmsl in **latest available** forecast using standard and hourly cycling

Advantages stemming from timely use of obs (global)

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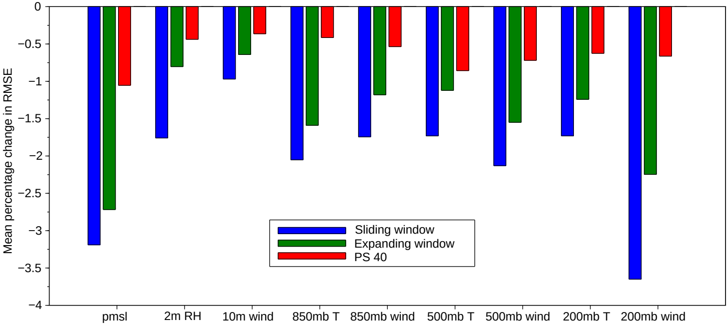

Figure: Percentage change in RMS forecast - obs error in latest available forecast meaned over forecast ranges of 1-33 hours, comparing traditional cycling with RUC Method 1 run hourly (blue) and RUC Method 2 run bi-hourly (green), and PS40-OS39 (red)

Advantages stemming from timely use of obs (LAM)

- Advantages stemming from timely use of obs:
  - Improve lateral boundary conditions for limited area models

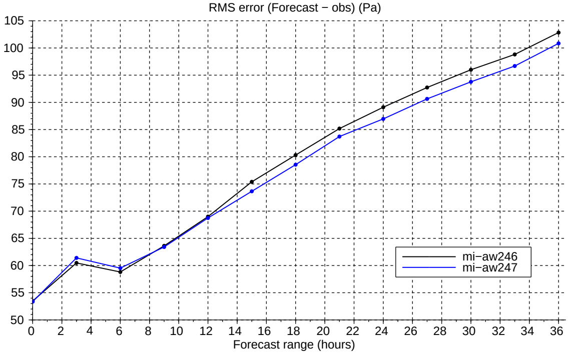

Figure: RMS forecast - obs for pmsl for UKV trial ingesting standard LBCs (black) and UKV trial ingesting LBCs from global run with Method 2 RUC (blue).

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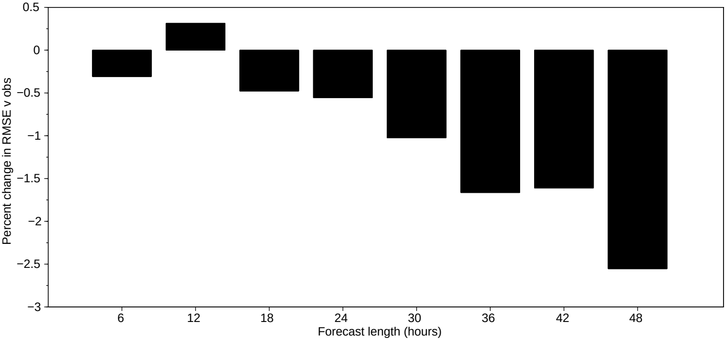

Figure: Percentage change in 10m wind RMS error, UKV trial ingesting LBCs from RUC cf standard LBCs

Concluding remarks

- Seek to assimilate obs a.s.a.p after receipt
- Most obs are received after delay of 1-4 hours
- For large-scale NWP systems choice between expanding DA windows and sliding DA windows
- Main practical advantage is in 'cutting off corners' in plot of error v forecast length, worth perhaps $\sim$2 years development
- Some benefit for LAMs in using timely LBCs
- In sliding window case currently have to hybridise with traditional cycling, due to limitations of fixed B
- Would like to remove need for hybridisation to realize other potential benefits (smaller increments, parallelism in time)

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