

Ensemble 4DVAR data assimilation in a dynamically challenging coastal ocean environment

IVO PASMANS

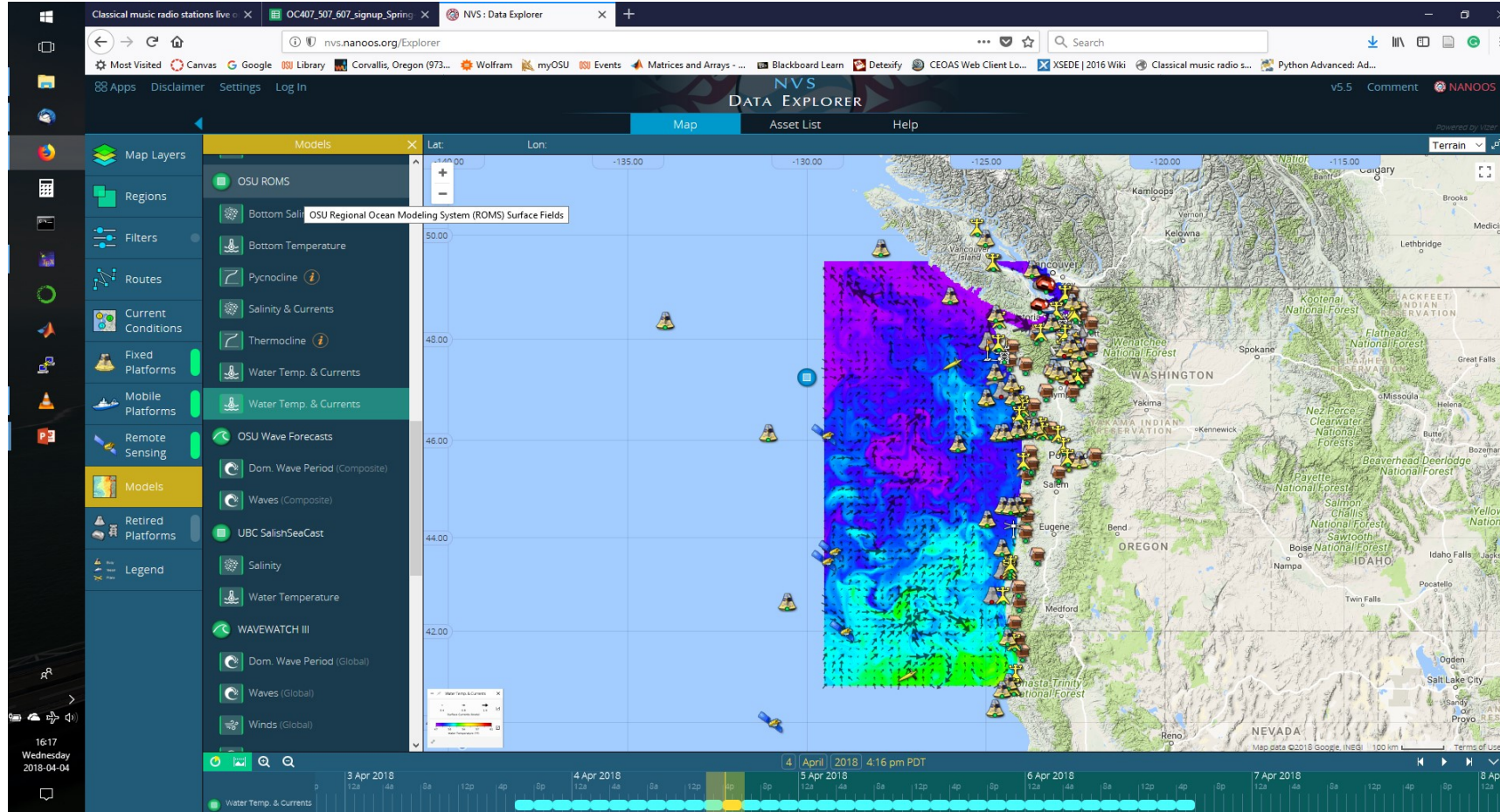
ALEXANDER KURAPOV

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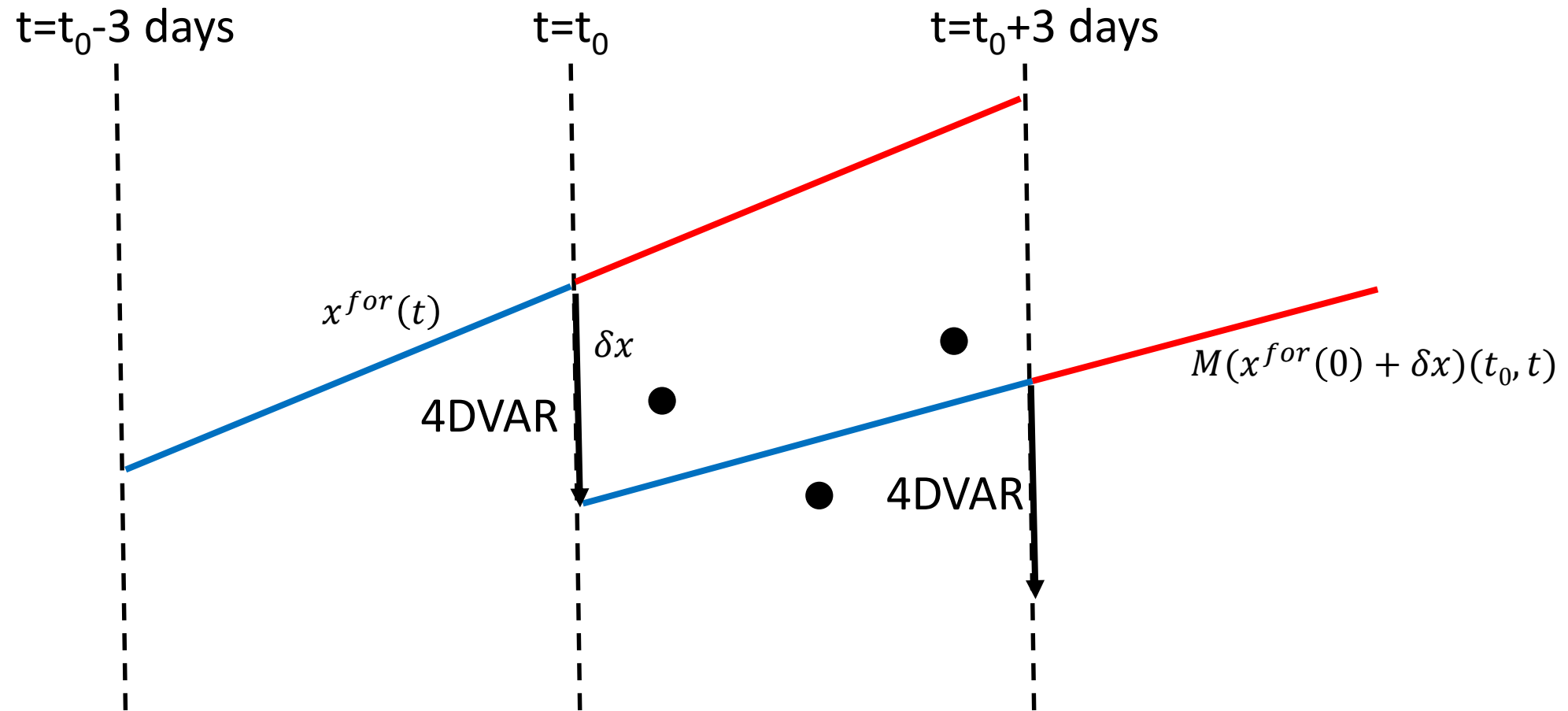


Motivation

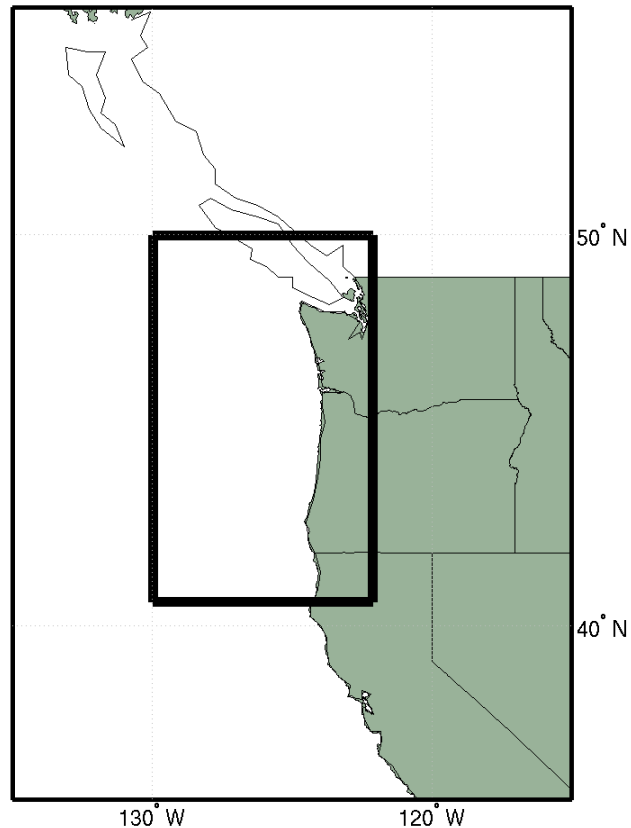
NANOOS model: ingria.coas.oregonstate.edu/rtdav/
Developed by: L. Erofeeva, A. Kurapov, I. Pasmans and P. Yu



Current forecasting system



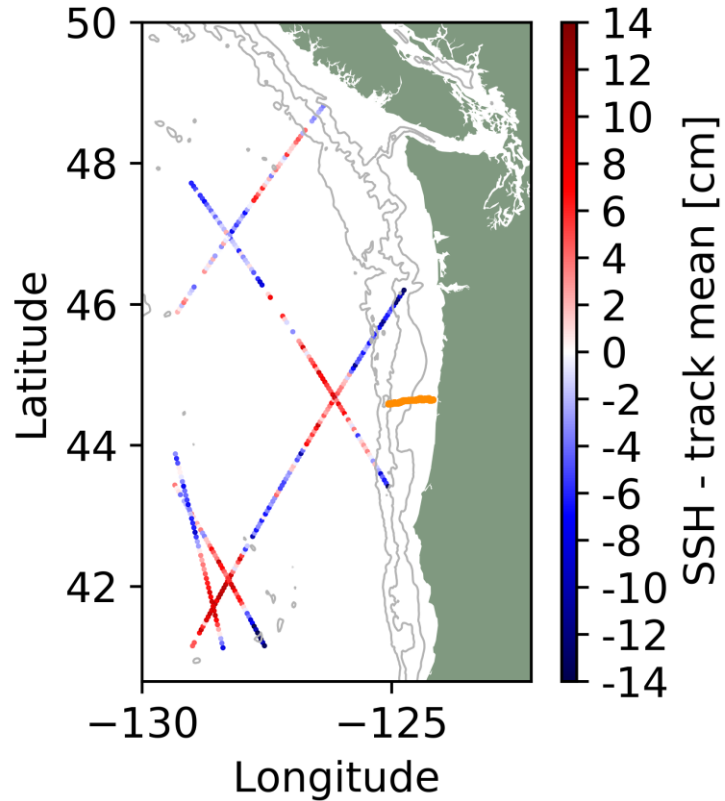
Nonlinear model



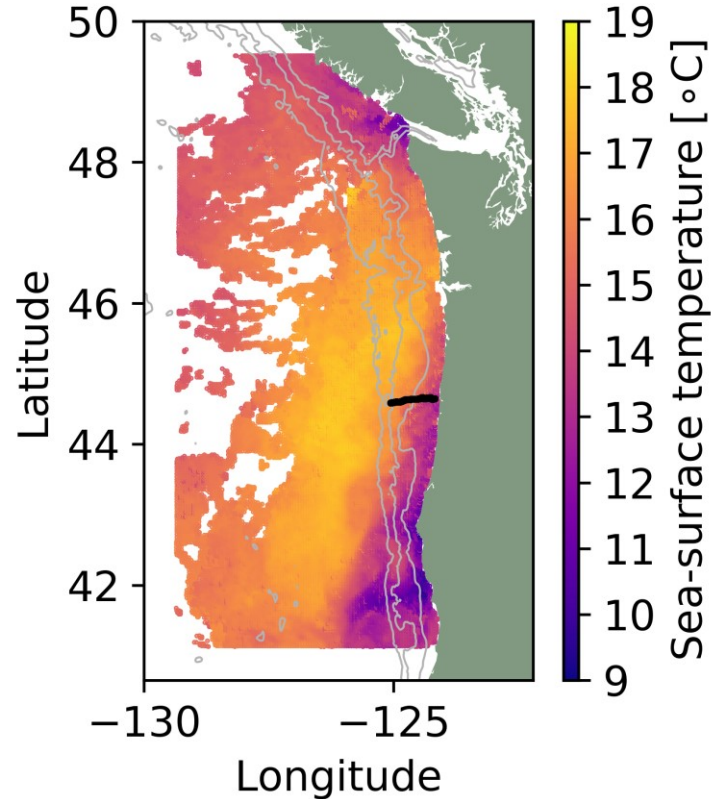
- Kim et al., 2014
- ROMS model
- 3D nonlinear hydrostatic Boussinesq model
- 2km resolution Arakaw-C grid
- 40 terrain following layers
- North American Model (NAM) wind forcing
- HYCOM boundary conditions
- TOPEX tides ad boundaries (Egbert et al., 1994; Egbert and Erofeeva, 2002)

Assimilated observations

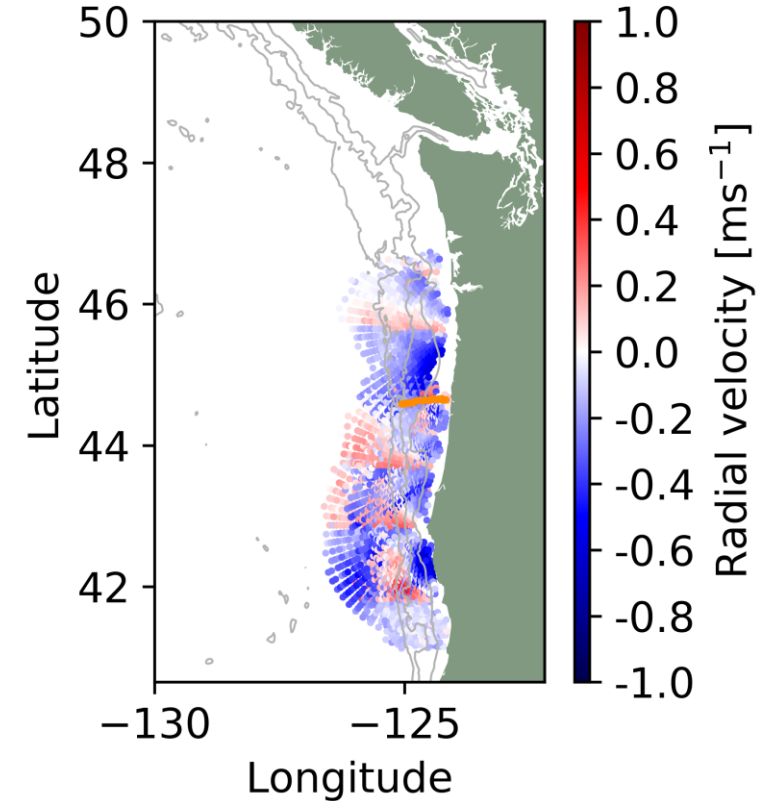
Satellite altimetry (Jason, Cryosat, Envisat)



GOES satellite sea-surface temperature



Radial high-frequency radar sea-surface currents (P. Kosro)

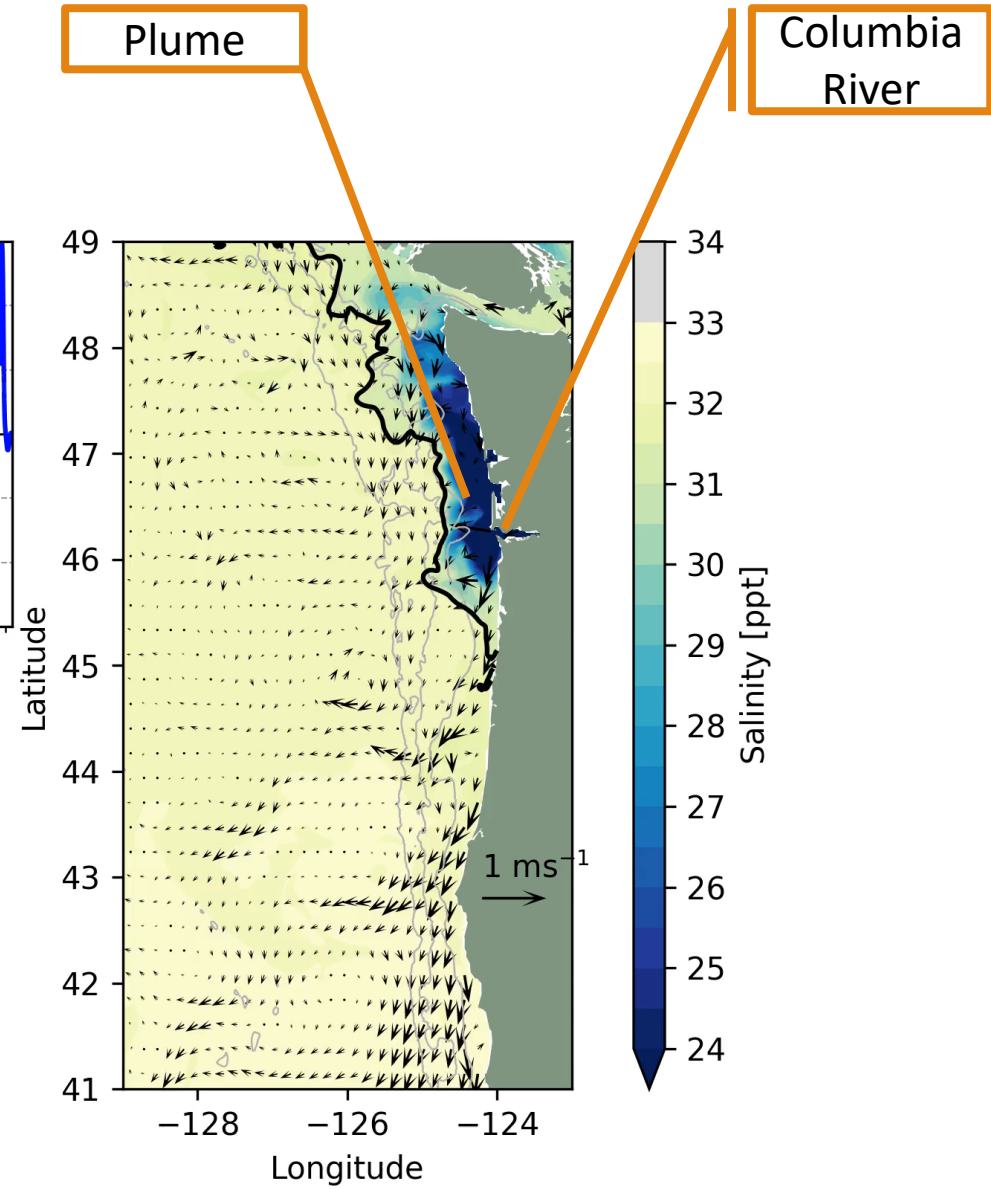
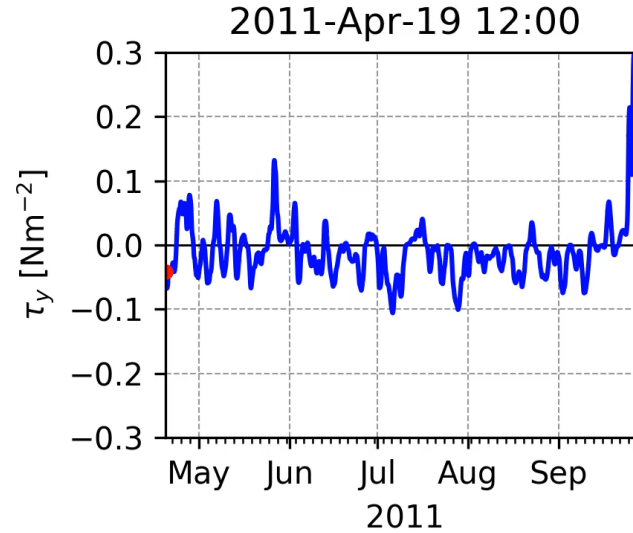


Background error covariance **B**

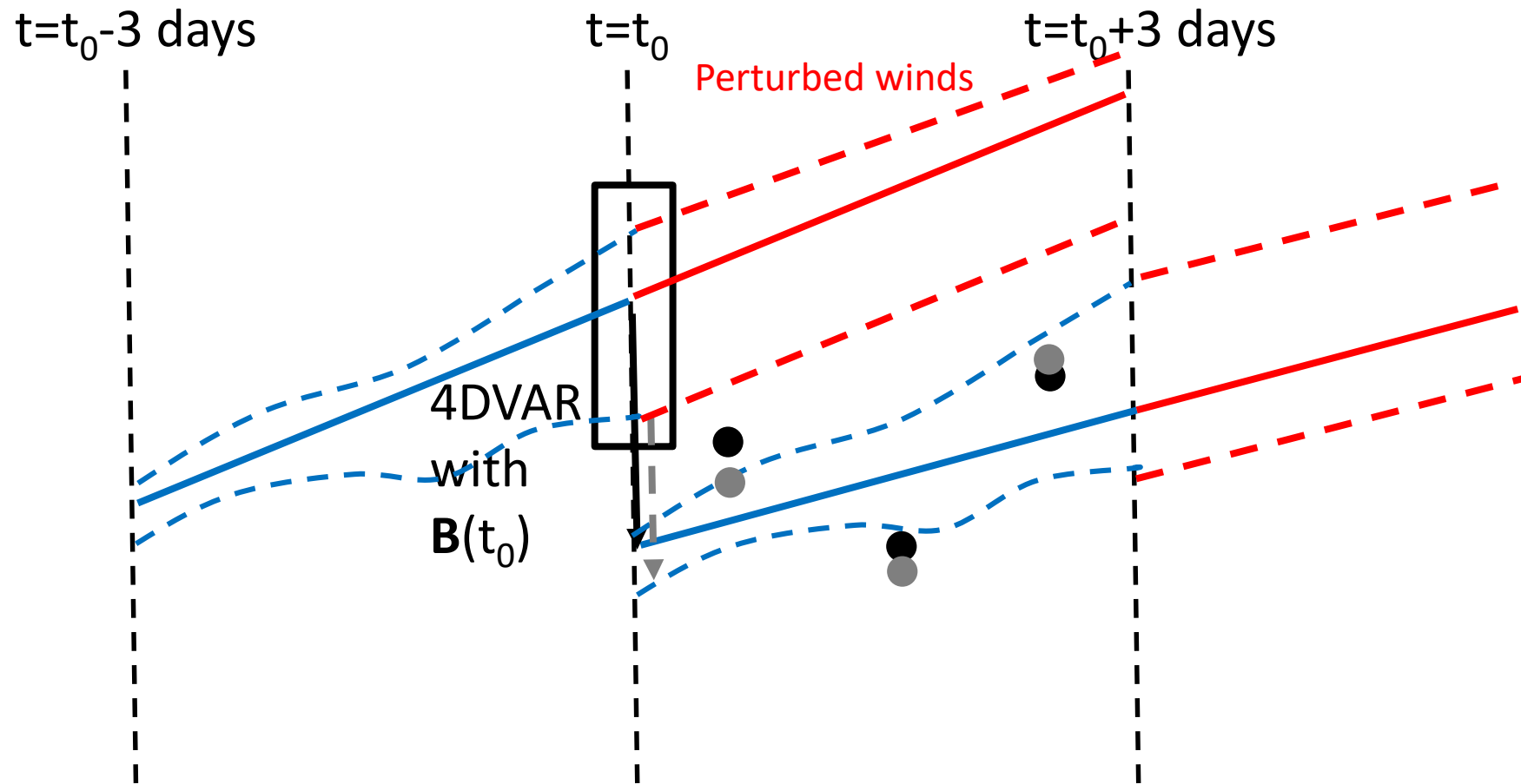
- 4DVAR performed by the in-house developed AVRORA code
- Balance operator covariance (Weaver et al., 2005)
 - Simple, fixed T,S error relation ($\delta S = -\alpha\delta T$, $\alpha = 0.16 \frac{psu}{C^\circ}$) from T,S-diagram
 - Linear equation of state
 - Thermal wind balance
 - No correction to depth-integrated transport
- Static in time.

Regional dynamics

- Strongly variable on regional and short time scales
- Southerly winds:
 - River plume flows northward
- Northerly winds:
 - River plume turns south
 - Upwelling deep ocean water ($\rho \geq 1026.5 \text{ kgm}^{-3}$)
 - Upwelling moves plume in offshore direction
- Time scale system: 2-10 days (Hickey et al., 1998)



E4DVAR method



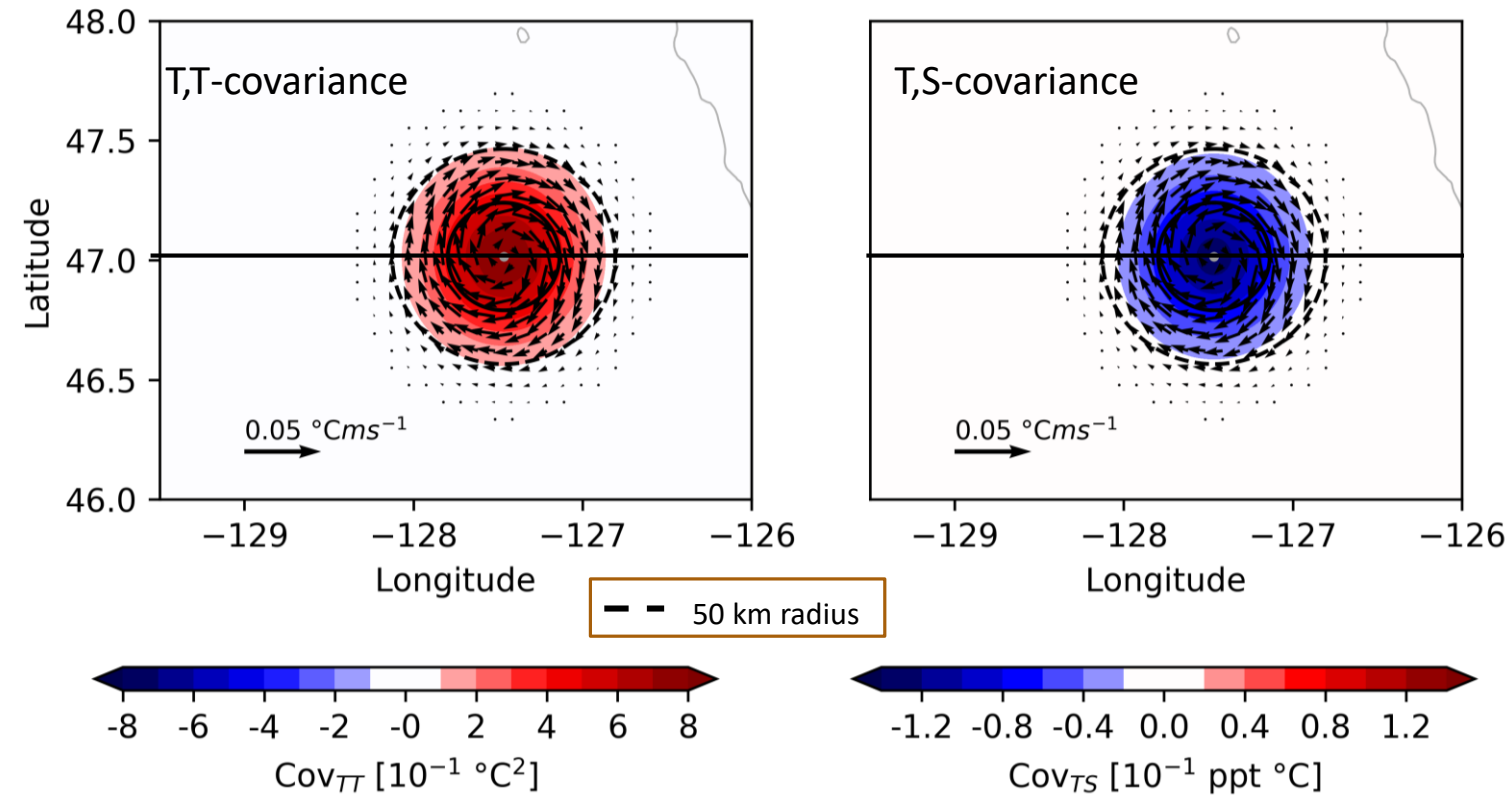
- Perturbed observations for ensemble members. (Houtekamer and Mitchell, 1998).
- Monte-Carlo localization (Pasmans and Kurapov, 2017).
- Restricted B-conjugate gradient (Gürol et al., 2014) preconditioning.
- Parallel expansion of the search space in which an iterative solution is sought.
- Lower-rank approximation inverse reused for ensemble members (similar to S-EVIL, Auligné et al., 2016).

Outline

- Compare covariances from the balance operator **B** with ensemble **B**.
 - Compare model result with respect to
 - Assimilated observations
 - Independent T,S-buoy observations
 - Glider T,S-relation
 - Present salinity constraining scheme.
-
- Experiments 19 April 2011-1 October 2011:
 - No data assimilation (*No DA*)
 - **B** constructed from an ensemble (*Ens*)
 - **B** constructed from an ensemble with a scheme controlling salinity corrections (*Ens-con*)
 - **B** constructed using the balance operator covariance with a scheme controlling salinity corrections (*Bal-con*)

Open ocean balance operator covariance

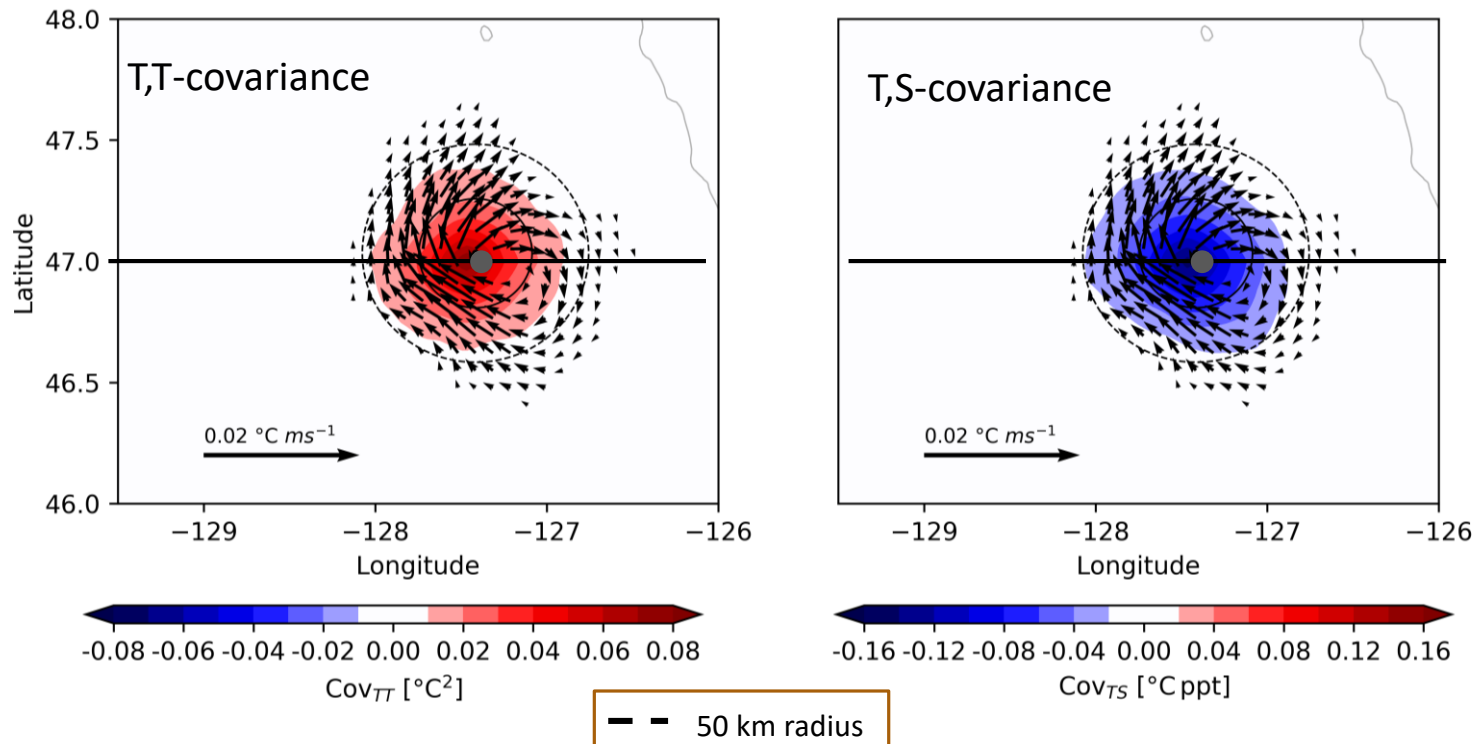
Balance operator SST, -covariances



Ensemble covariance: open ocean

On average localized ensemble covariance reproduces the balance operator covariance, but with a T-variance a factor 10 smaller.

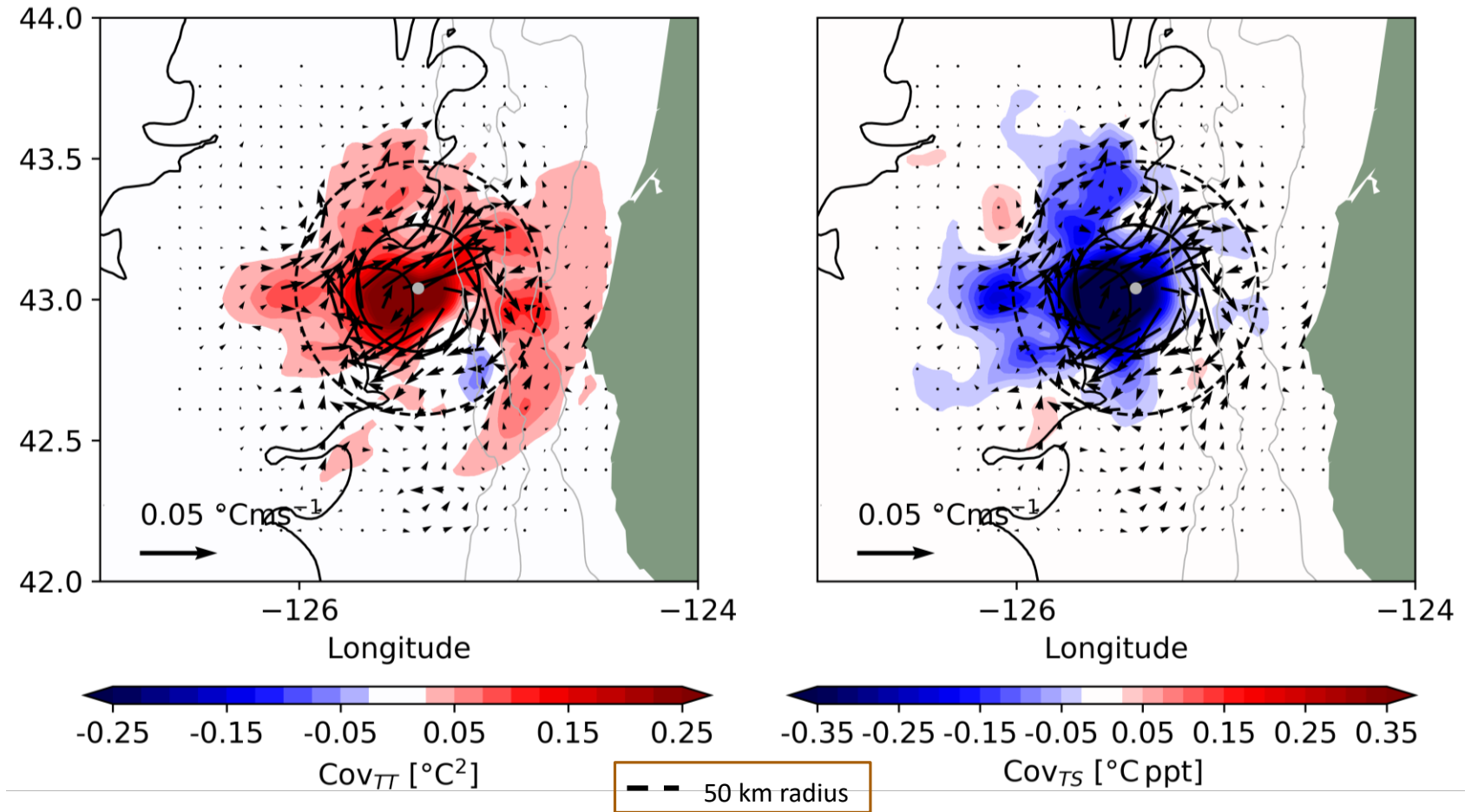
Averaged localized SST, · -covariances



Ensemble covariance: fronts

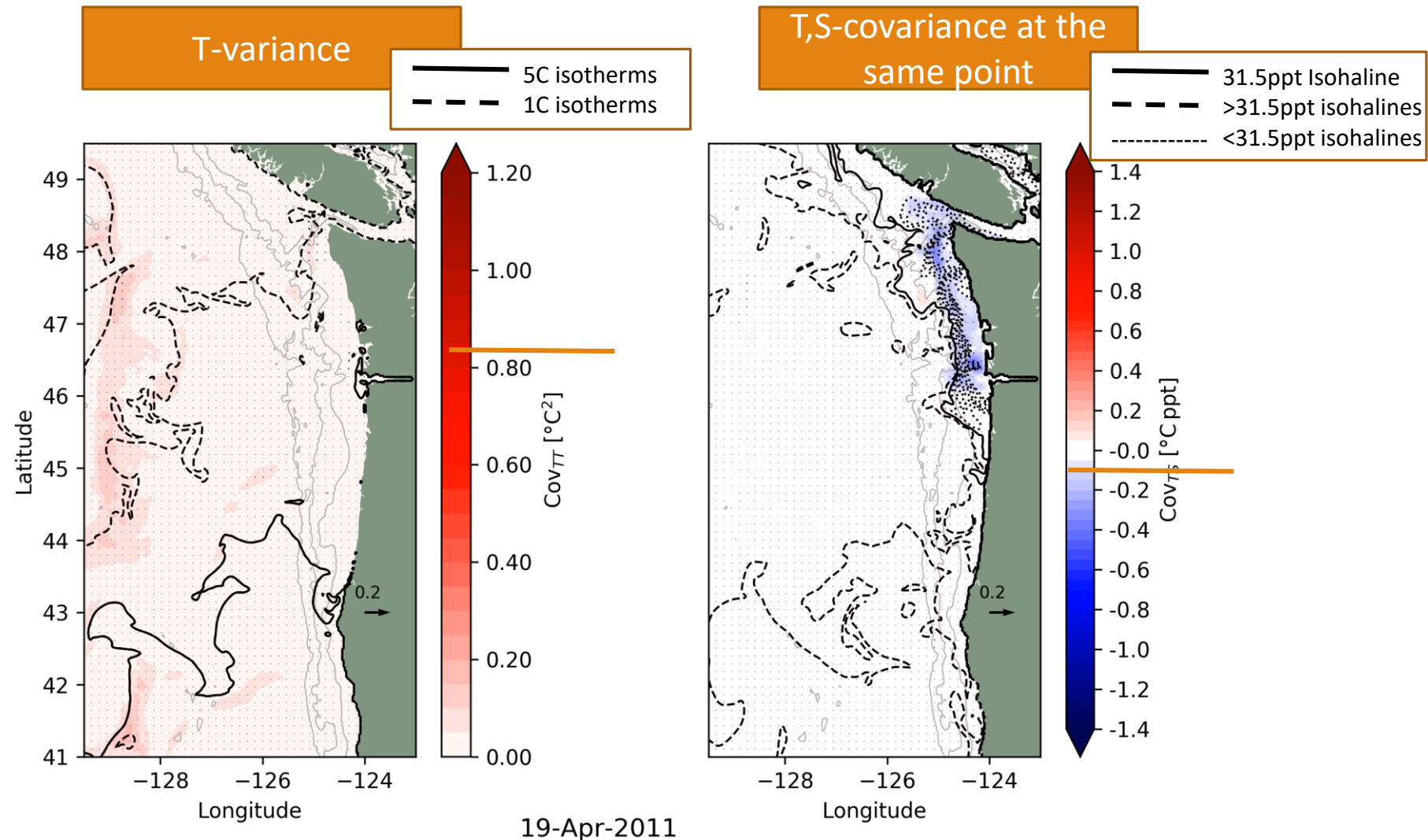
- Temperature correlated with cross-front velocities instead of anti-cyclonic velocities.

Localized SST, -ensemble covariance 9 July 2011



Ensemble surface (co)variance

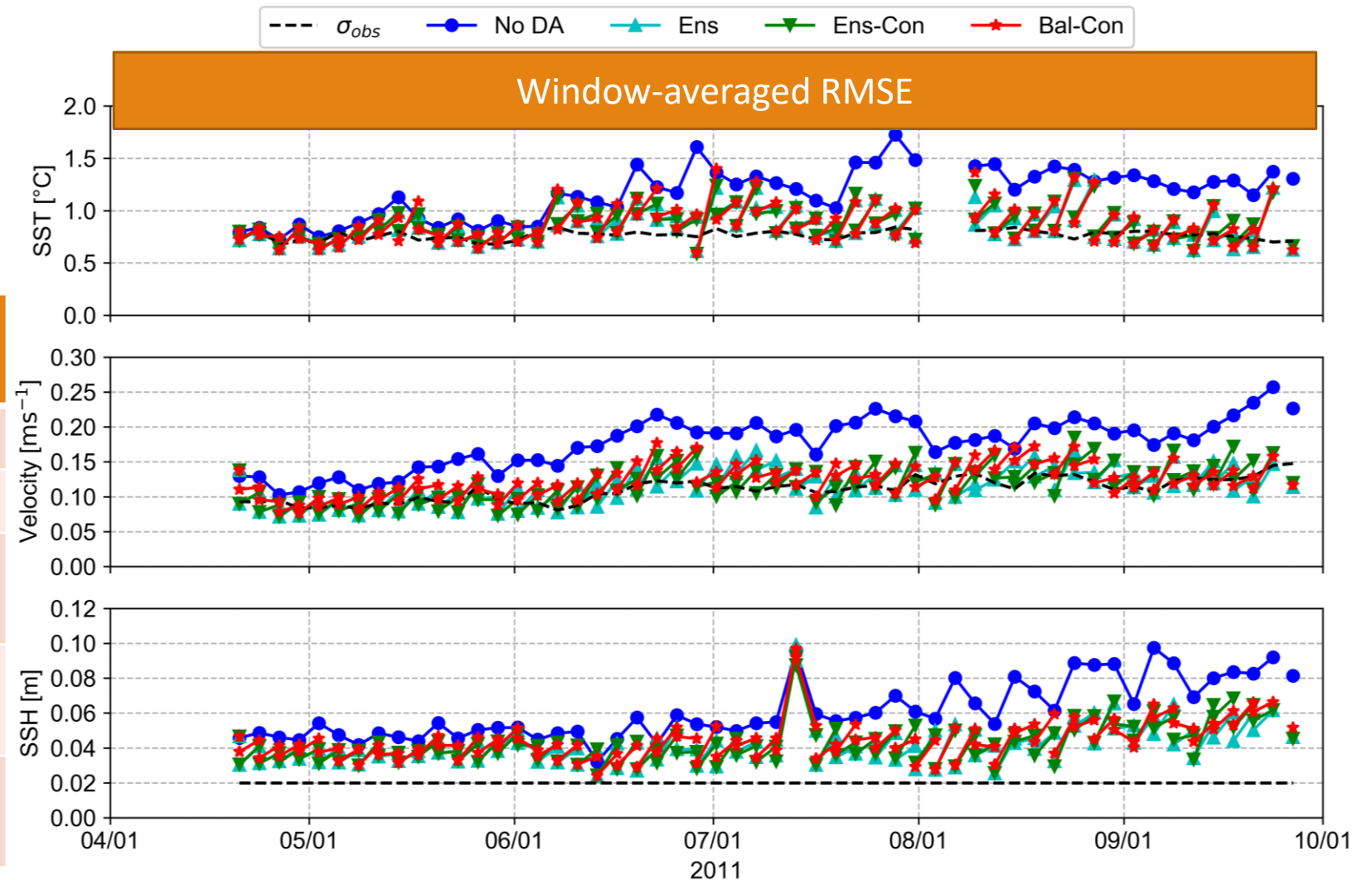
- General magnitude T-variance and T,S-covariance smaller than balance operator equivalents (0.81°C^2 , $-0.13 \text{ ppt }^{\circ}\text{C}$)
- Large magnitudes near fronts.
- Large magnitudes T and S at different locations.



Analysis-forecast RMSE per window

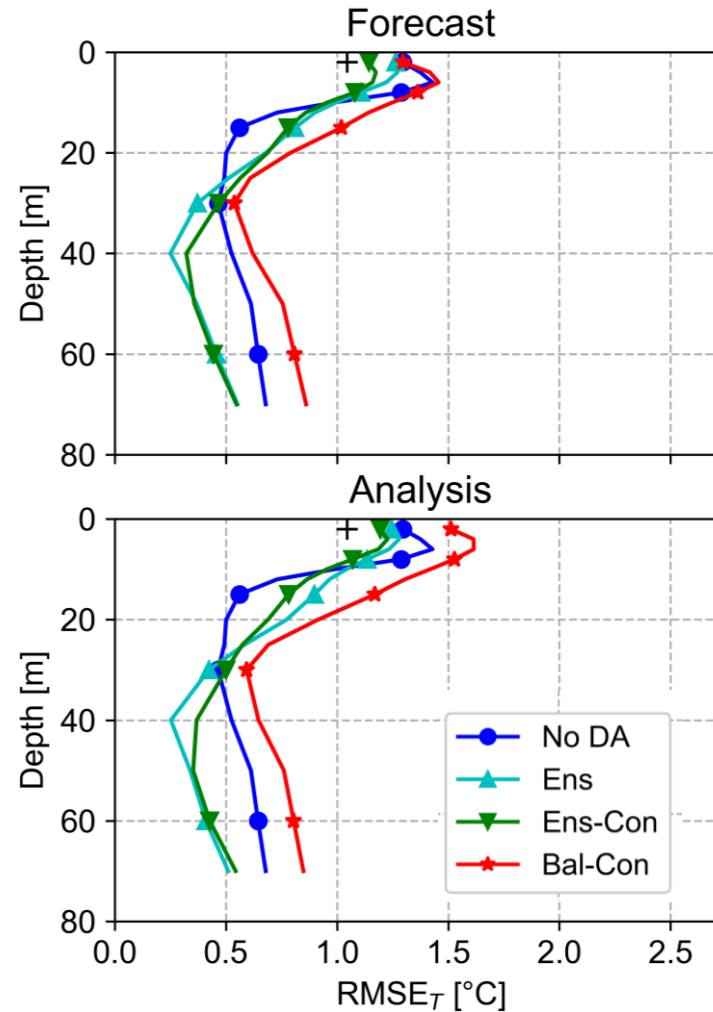
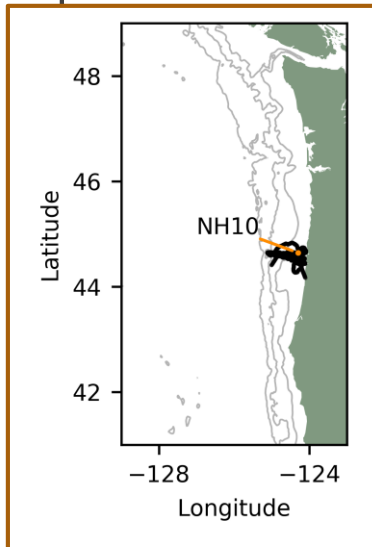
- DA reduces errors in analyses and forecasts
- E4DVAR slightly outperforms Balance Operator covariance.

| Experiment | SST [C] | u [m/s] | SSH [m] |
|------------|----------------|----------------|----------------|
| σ | 0.768 | 0.112 | 0.020 |
| No DA | 1.189 | 0.180 | 0.063 |
| B ens | 0.775 0.963 | 0.104 0.131 | 0.038 0.049 |
| B ens-con | 0.777 0.970 | 0.104 0.133 | 0.039 0.049 |
| B bal-con | 0.763 1.001 | 0.114 0.137 | 0.042 0.050 |



NH10 comparison

- E4DVAR as accurate as satellite SST (+) near surface.
- E4DVAR more accurate below the surface layer.
- Balance Operator performance deteriorates if no satellite SST observations near NH10 are present.



Average RMSE 4/19-10/1

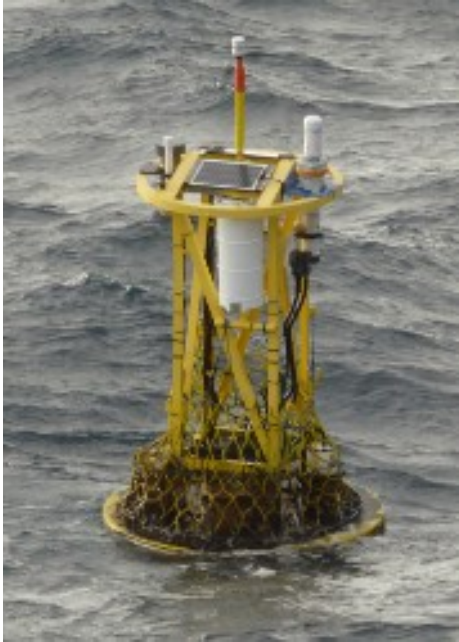
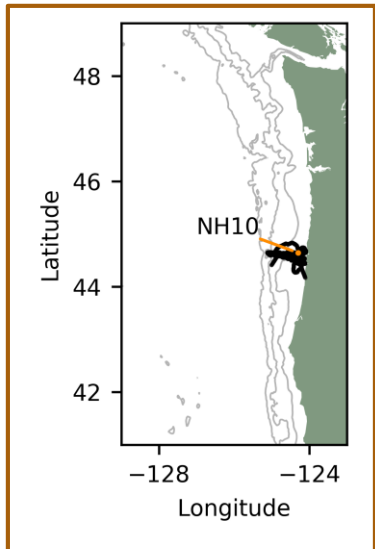


Image source: http://www.ndbc.noaa.gov/station_page.php?station=46094

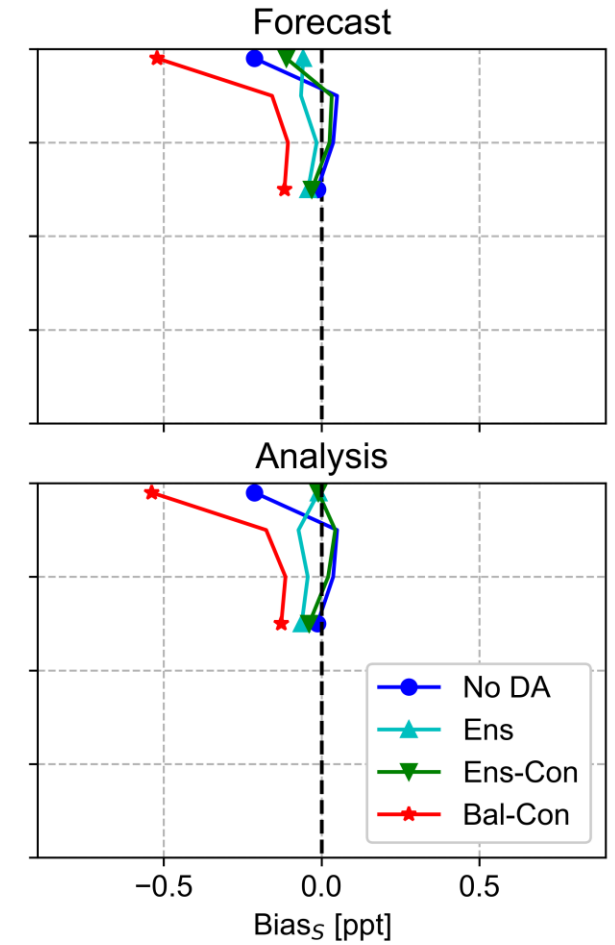
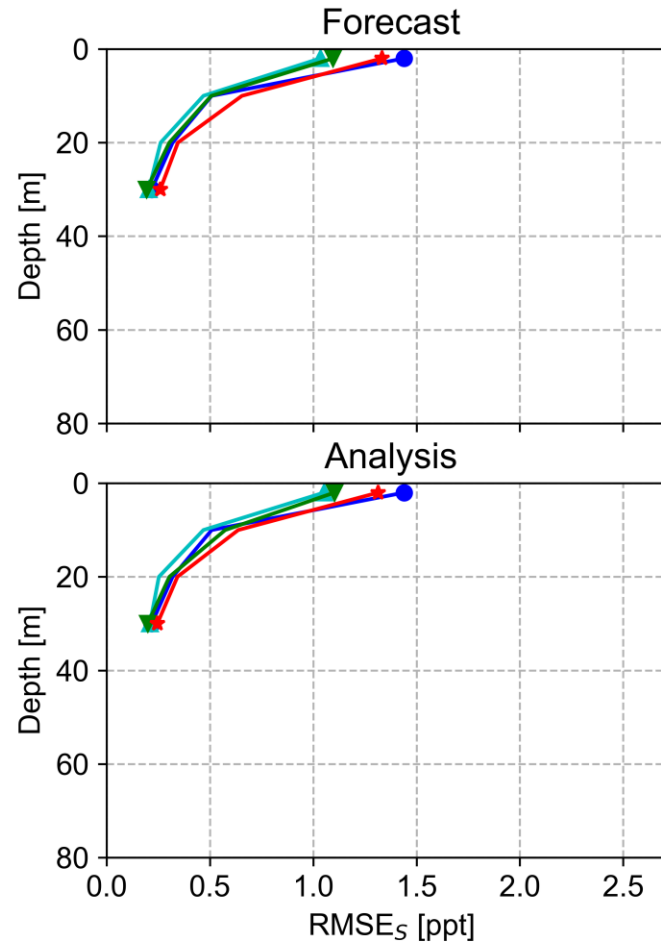
NH10 comparison

- E4DVAR produces more accurate results for salinity near the surface.



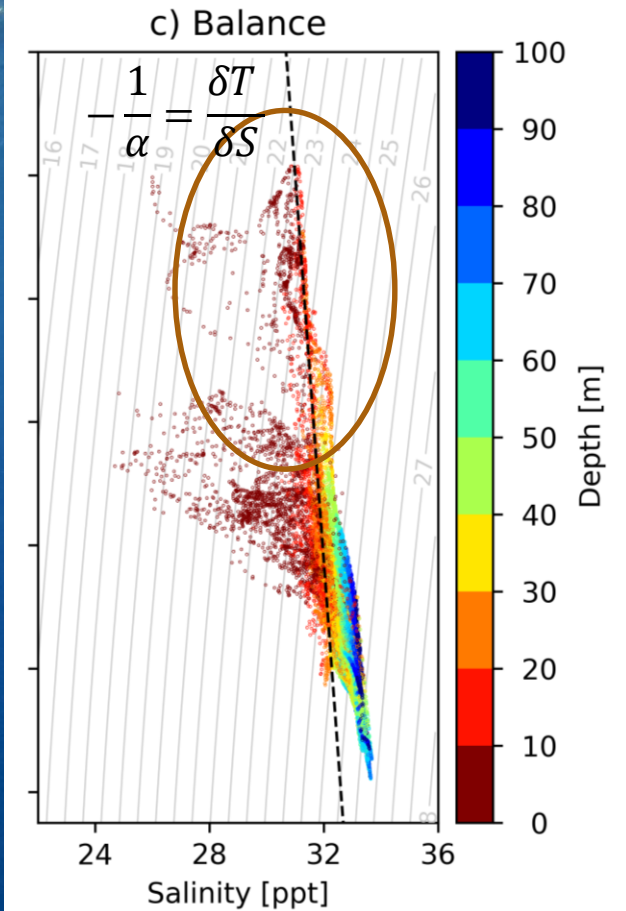
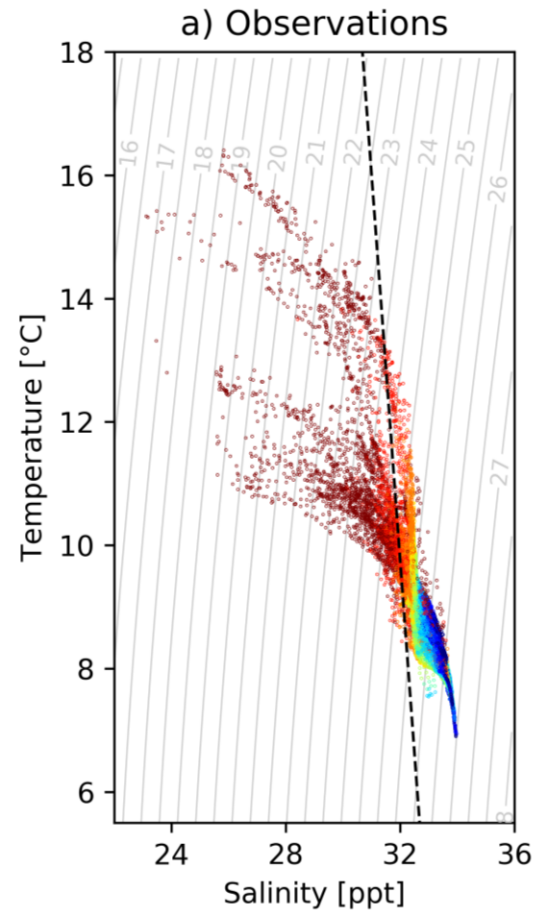
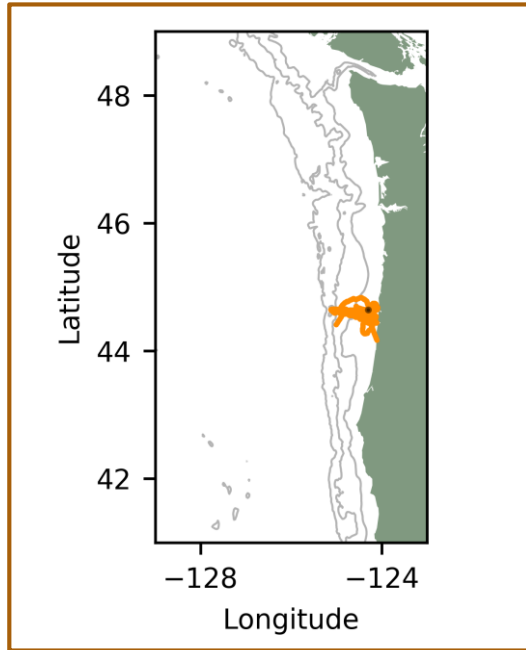
Average RMSE 4/19-10/1

Average bias 4/19-10/1



T,S-relation

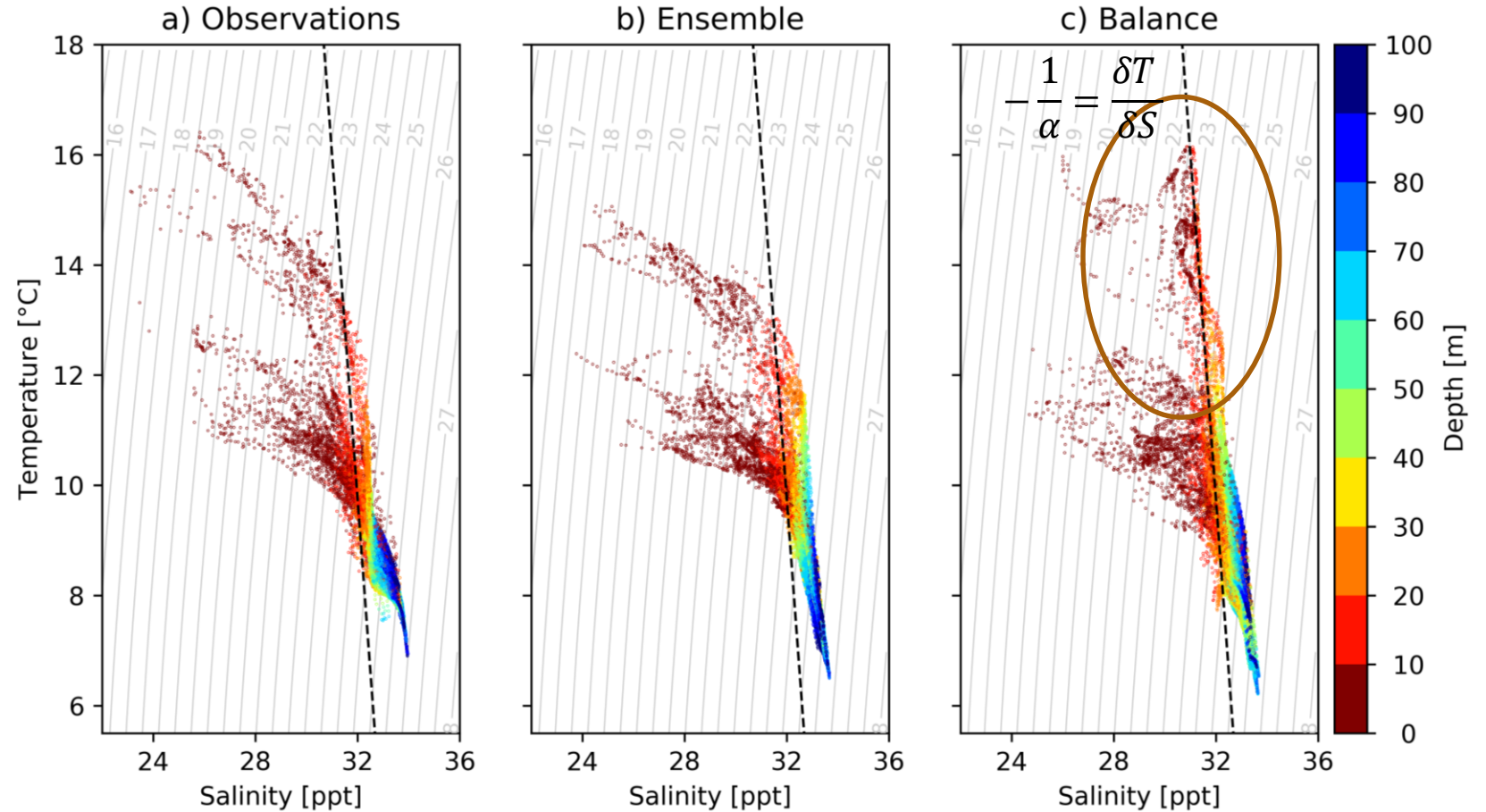
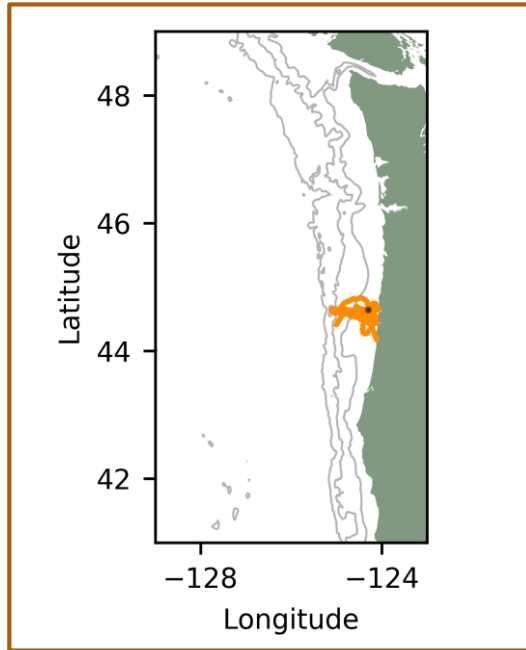
- Deformation T,S-relation using Balance Operator
- Using E4DVAR removes deformation



Forecasts 19 April 2011-30 June 2011

T,S-relation

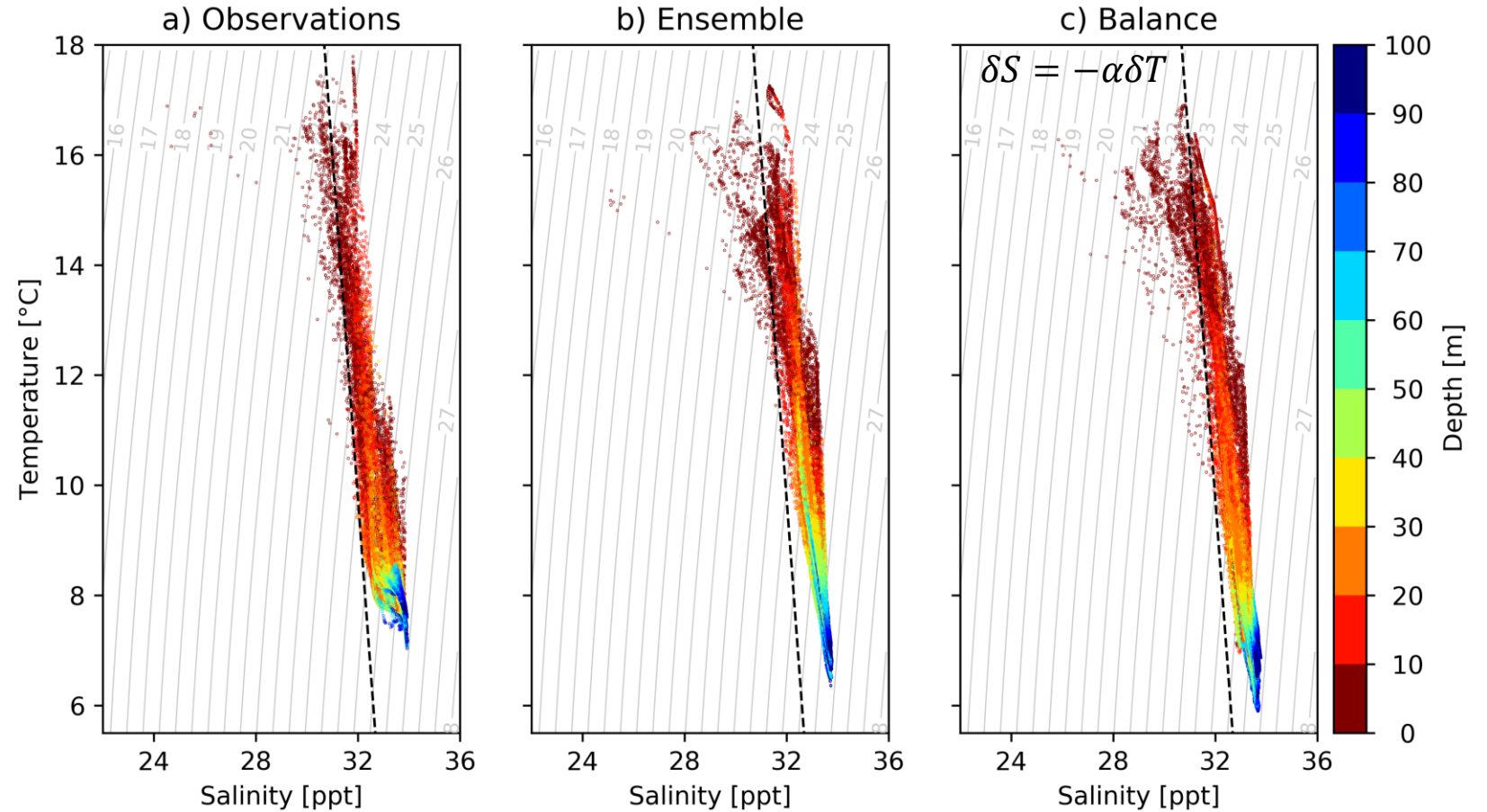
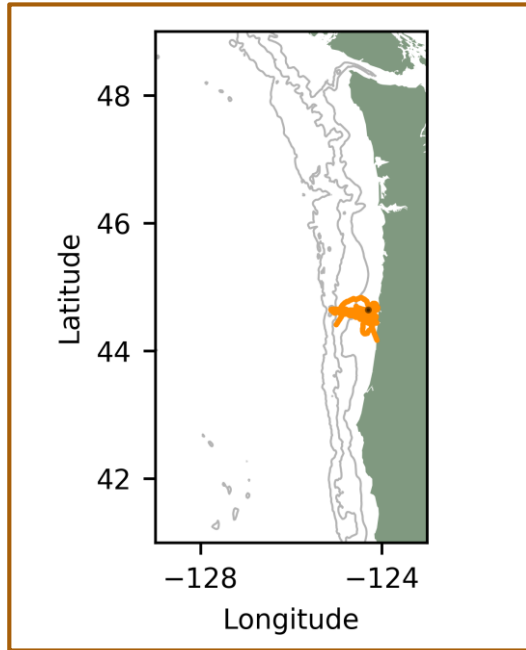
- Deformation T,S-relation using Balance Operator
- Using E4DVAR removes deformation



Forecasts 19 April 2011-30 June 2011

T,S-relation

- E4DVAR correctly captures T,S-relation when it reverts to $-0.16 \text{ ppt}/^\circ\text{C}$

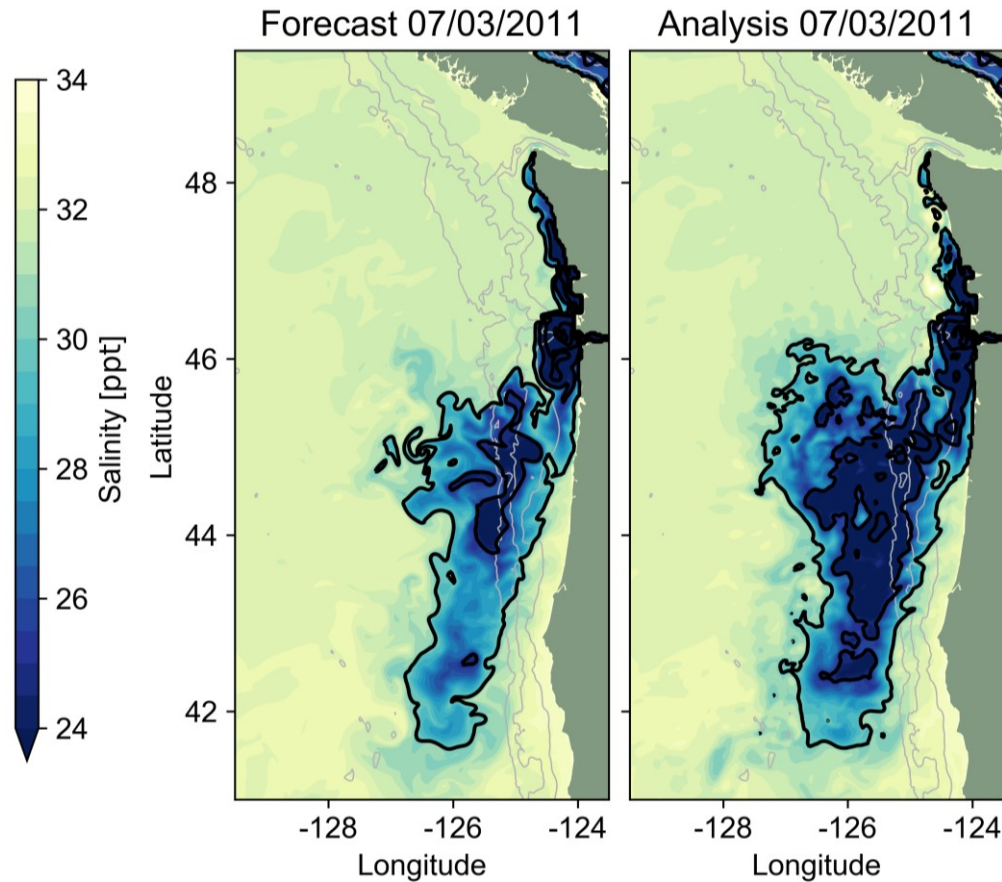


Forecasts 1 July 2011 – 1 October 2011

DA induces large corrections plume size

No constraint on salinity

- Assimilation with ensemble **B** can instantly increase/decrease the size of the plume → violation conservation salinity.



Salinity constraining scheme

- Divide domain up into a hierarchy of boxes of different sizes.
- Add penalty to the cost-function for DA induced changes in average salinity of each box.
- Large-scale changes in surface salinity cause the total salinity penalty to increase exponentially.

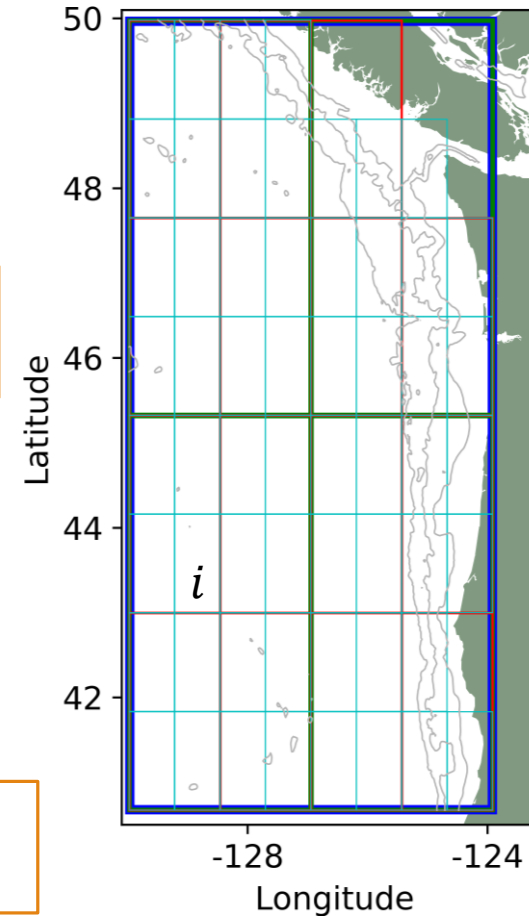
$$J' = J + \sum_i \frac{(\langle S \rangle_{ana,i} - \langle S \rangle_{for,i})^2}{\sigma_i^2}$$

$$\langle S \rangle_i = \frac{1}{A_i} \int_{A_i} S \, dA$$

Change average salinity due to DA

Weighting based on variability in *No DA*

Salinity boxes



Constraining affects DA changes plume

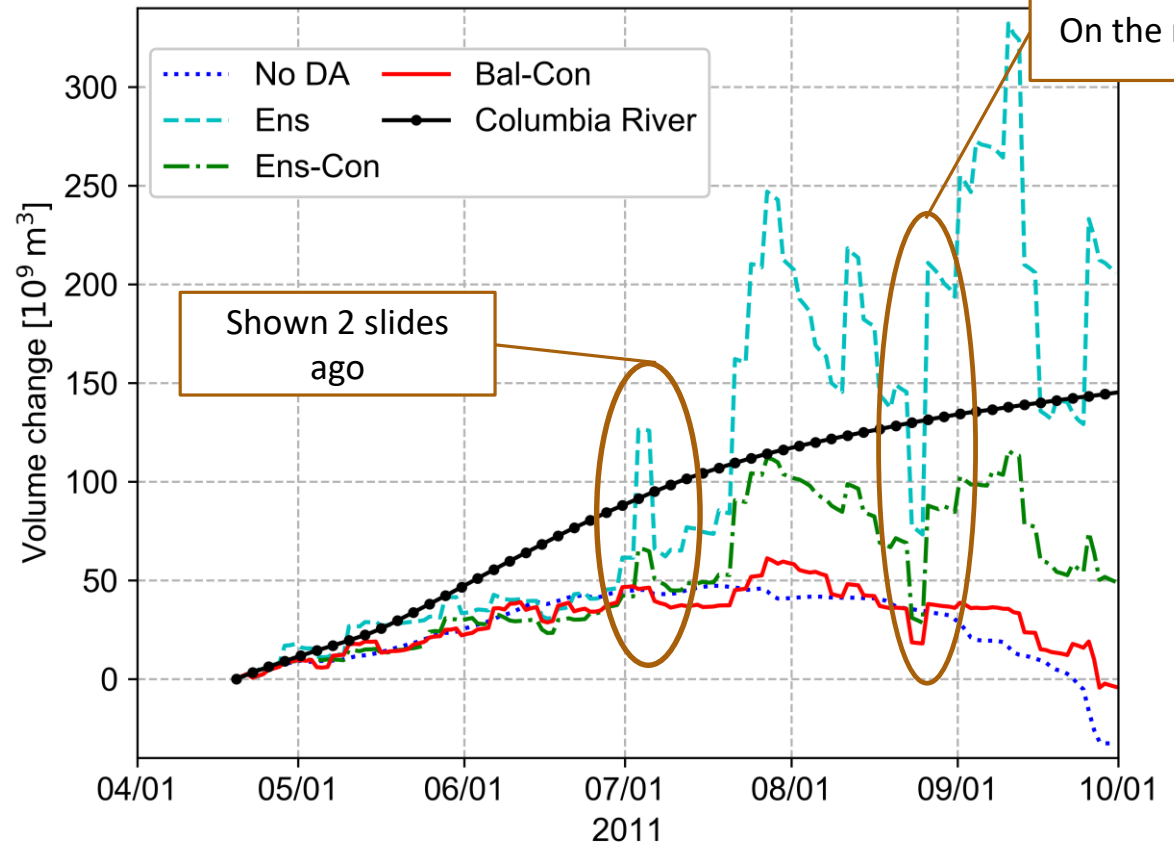
- Constraining salinity drastically reduces changes in plume water volume by data assimilation.
- Constraining does not completely eliminate the changes to plume water volume by DA.

Find plume water volume per grid cell by solving

$$S_{grid\ cell} V_{grid\ cell} = S_{fresh} V_{fresh} + S_{ocean} (V_{grid\ cell} - V_{fresh})$$

$$S_{fresh} = 0.3\ ppt, S_{ocean} = 32.2\ ppt$$

Plume water volume south of the Columbia River

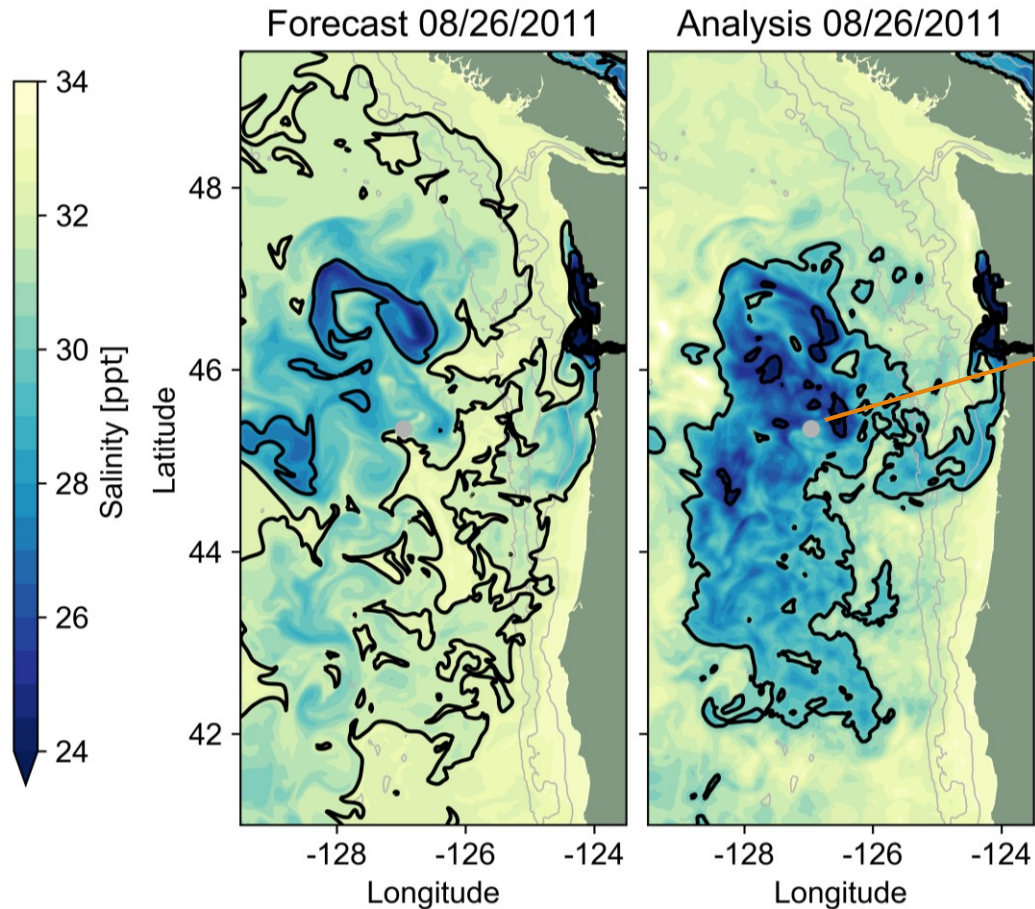


DA corrections are not physical.

No constraint on salinity

- Comparison with Argo float shows that the large DA corrections to salinity reduce surface salinity too much.

ARGO salinity profile 3 September



Ensemble covariance → too fresh

Constraining improves results

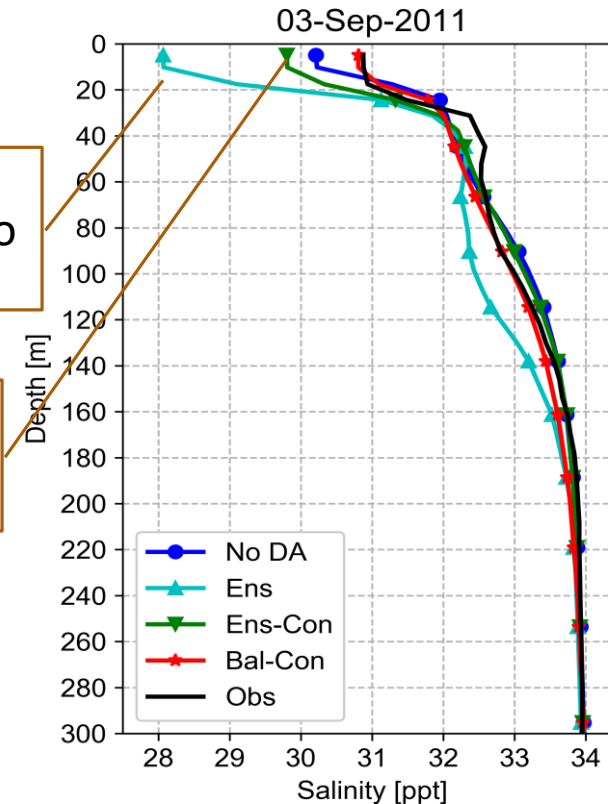
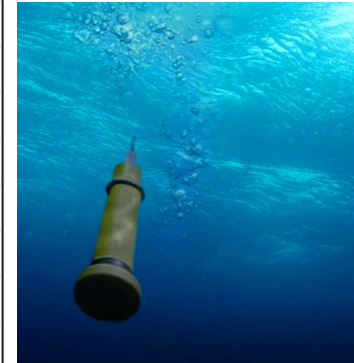


Image source:
https://samcharlesjones.files.wordpress.com/2015/07/argo_final2-copy.jpg



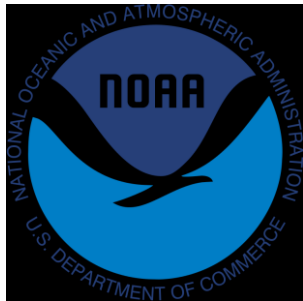
Conclusions

- In the presence of a river plume the currently used Balance Operator covariance is not a good approximation for the background error covariance.
- Use of an ensemble-based background error covariance has some benefits over the use of a Balance Operator based background error covariance:
 - Better representation of the T,S-relation along the Newport line.
 - Produces forecasts with lower RMSEs for NH10 and for satellite SST, SSH and HFR surface velocities.
- As a result of the large T,S-covariances in the ensemble covariance, data assimilation corrections to SST can generate unphysical changes in the size and salinity of the plume.
 - Implementing tracer conservation laws as (weak) constraint necessary in future studies.

Questions?

Acknowledgement:

This presentation was made possible thanks to financial support from NOAA Coastal Ocean Modeling Testbed (COMT) grant NA13NOS0120139, Integrated Ocean Observing System / Northwest Association of Networked Ocean Observing Systems (IOOS/NANOOS) grant NA16NOS0120019, the NASA SWOT Science Definition Team project grant NNX13AD89G. This work used the Extreme Science and Engineering Discovery Environment (XSEDE) under allocation TG-OCE160001, which is supported by National Science Foundation grant number ACI-1548562 and travel funding from the Workshop on Sensitivity Analysis and Data Assimilation in Meteorology and Oceanography.



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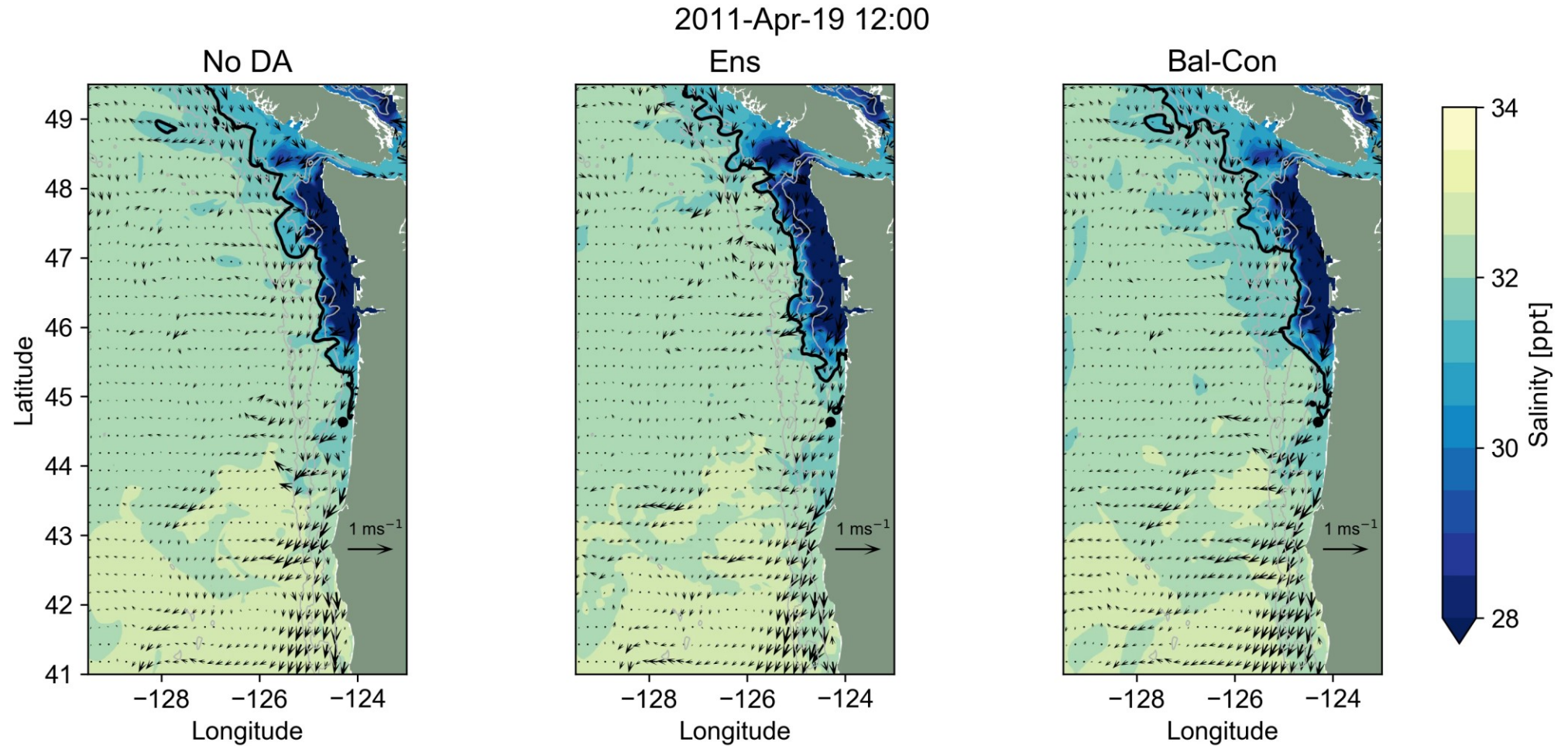


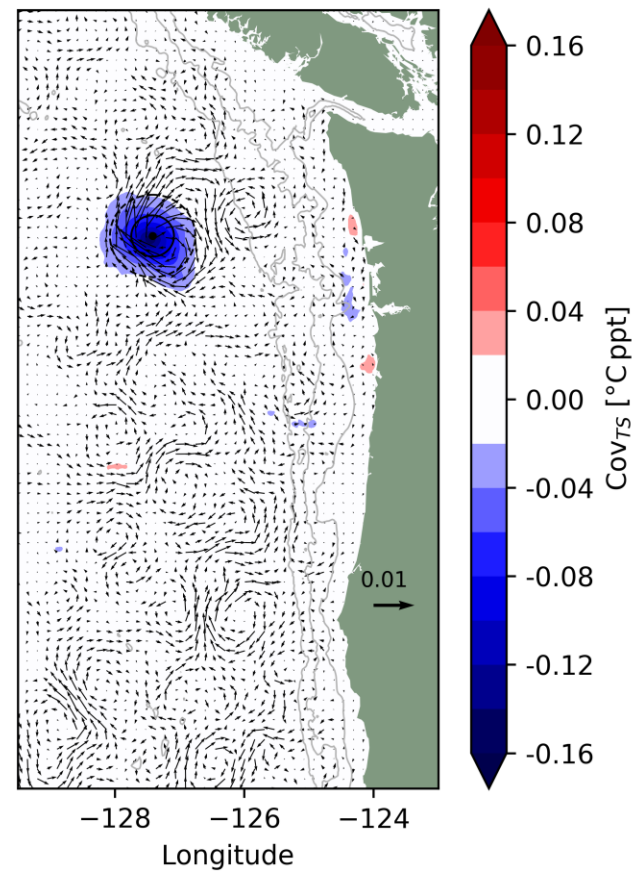
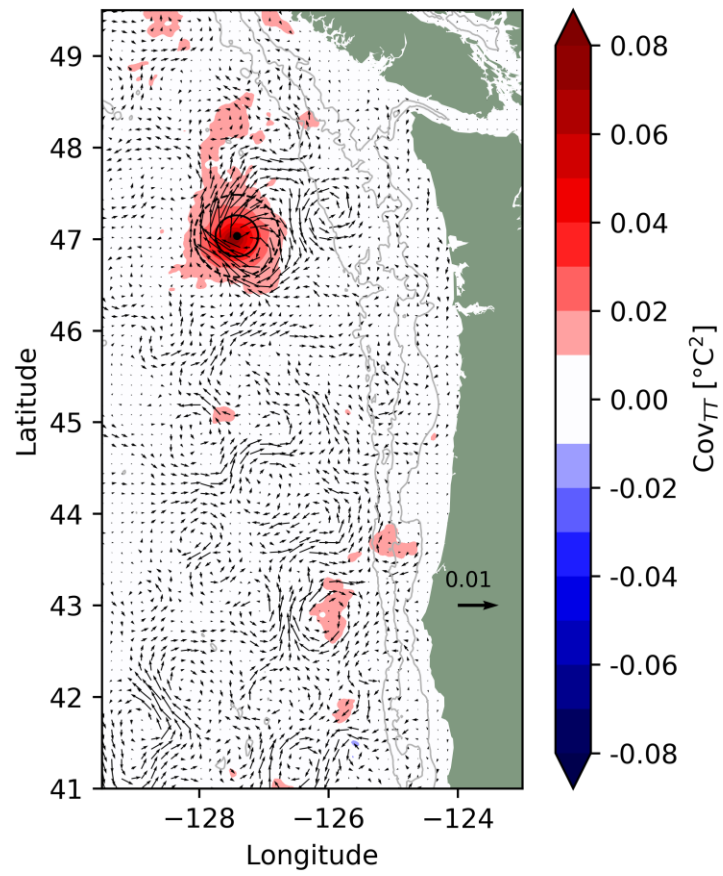
Oregon State University

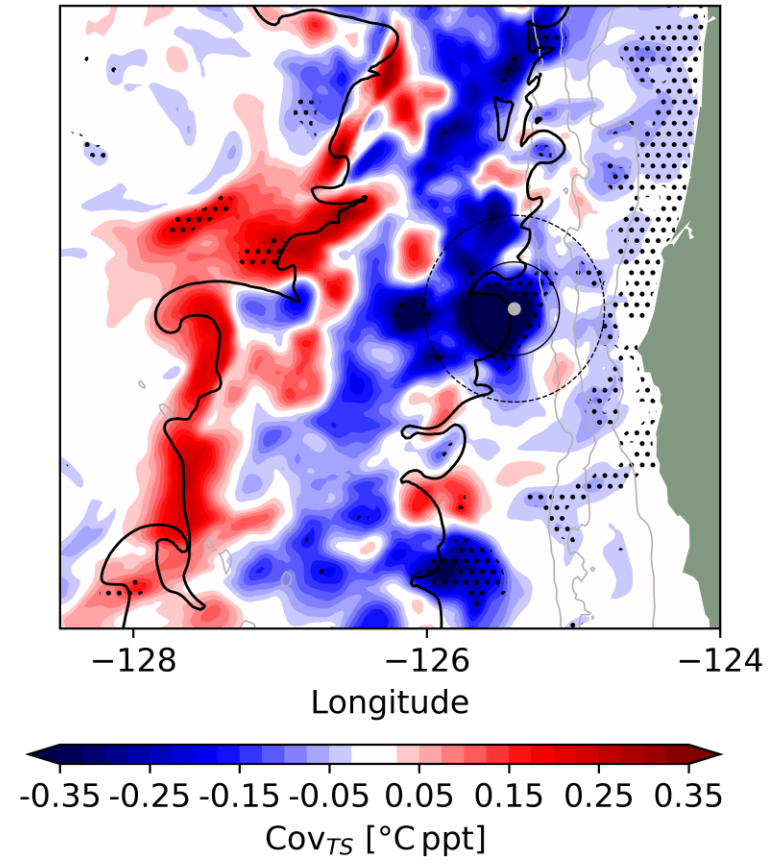
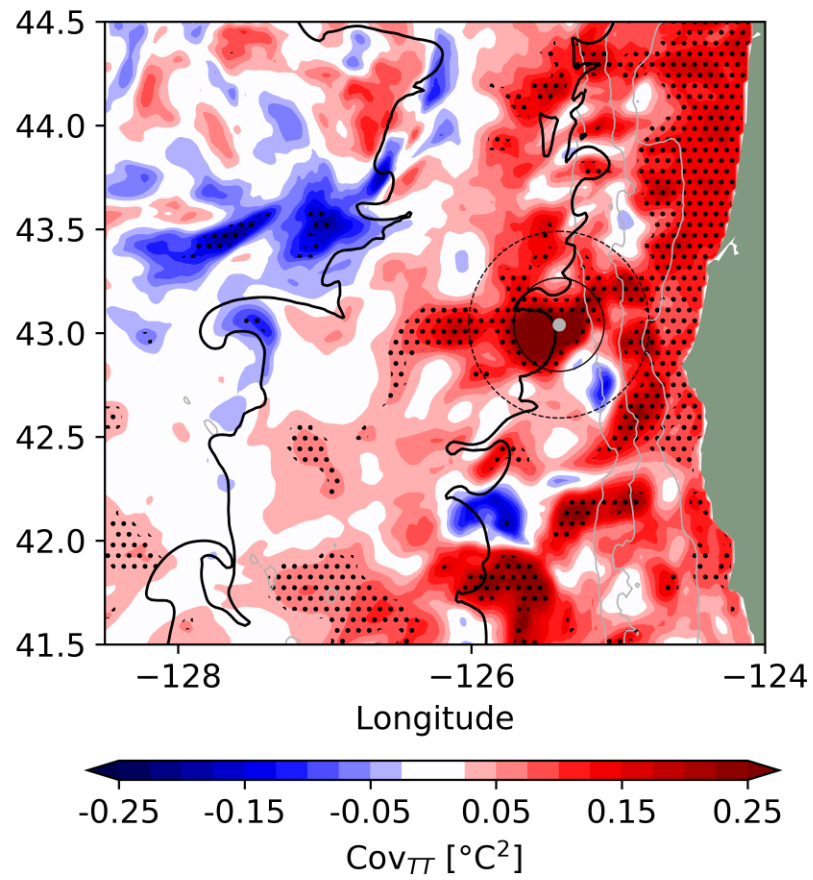
Sea-surface salinity

Sea-surface salinity from analyses

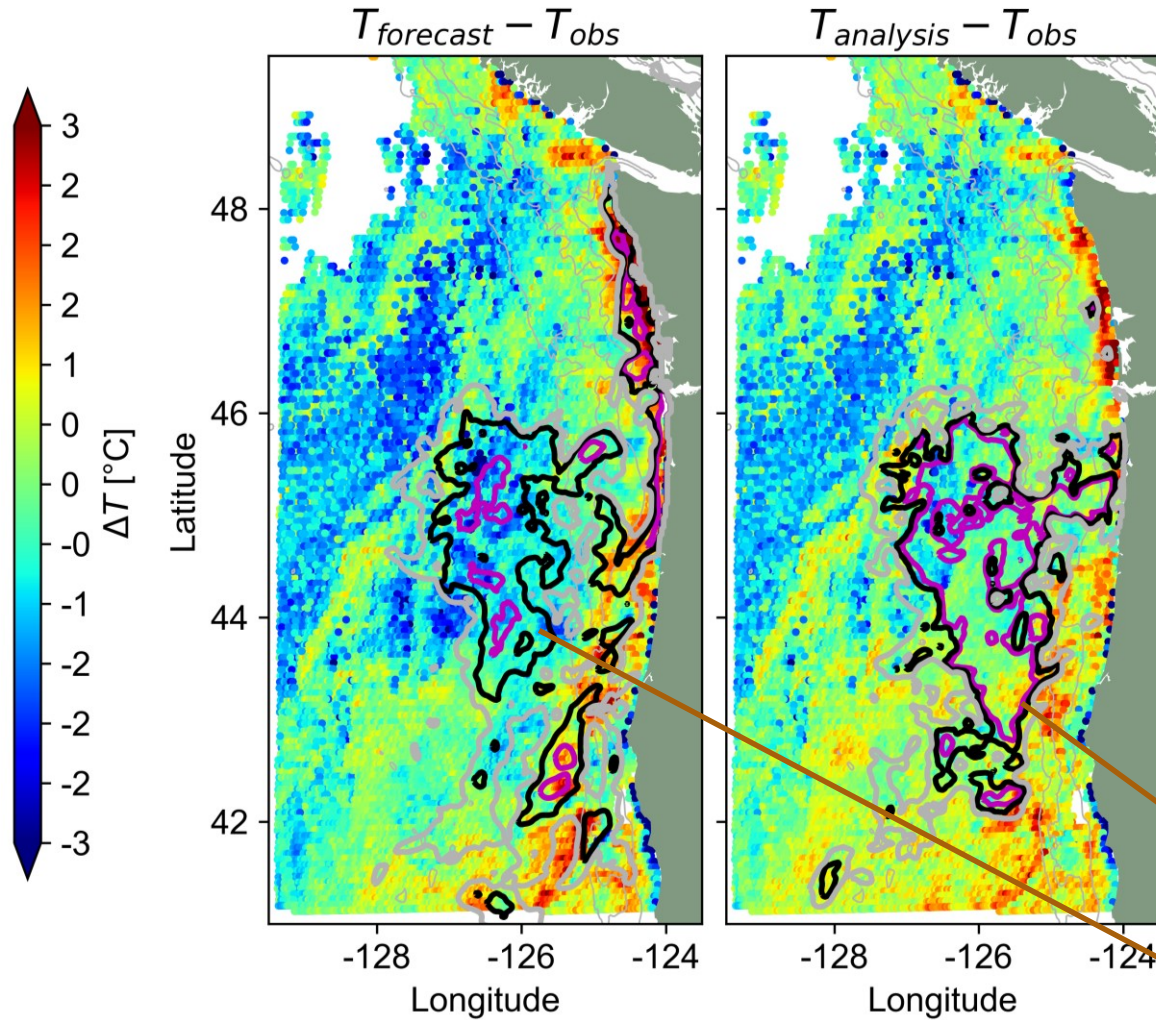
When using E4DVAR assimilation severely impacts the extend and salinity of the plume.







Reason plume size corrections



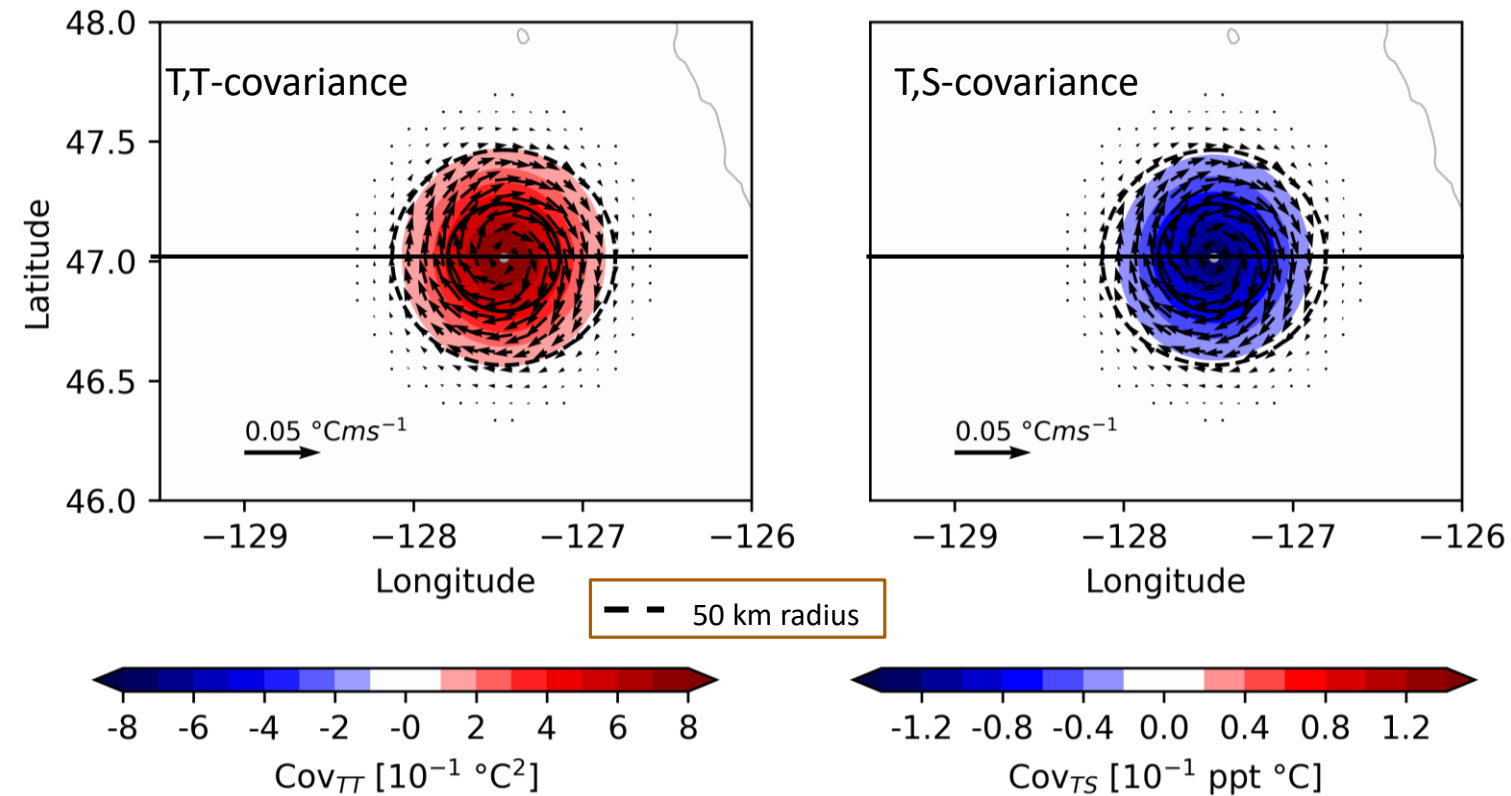
Large salinity corrections are caused by large corrections to SST in combination with the presence of large T,S-covariances.

DA correction to surface salinity (-1,-2,-3 ppt)

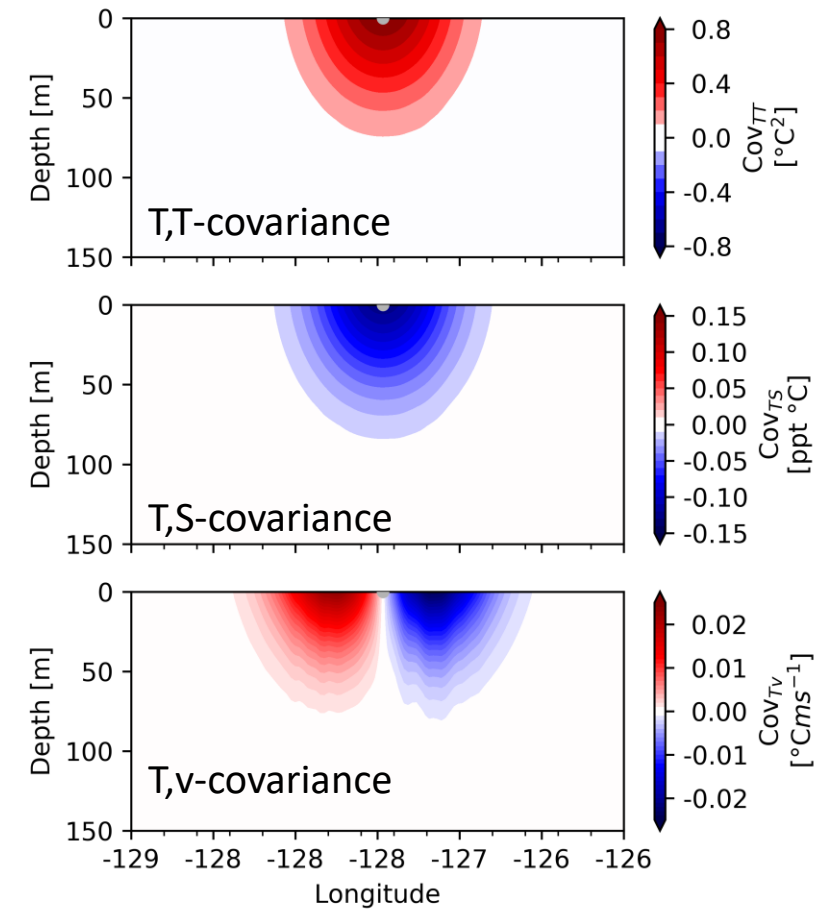
T,S-covariance (1x,2x,4x balance operator covariance)

Open ocean balance operator covariance

Balance operator SST, · -covariances



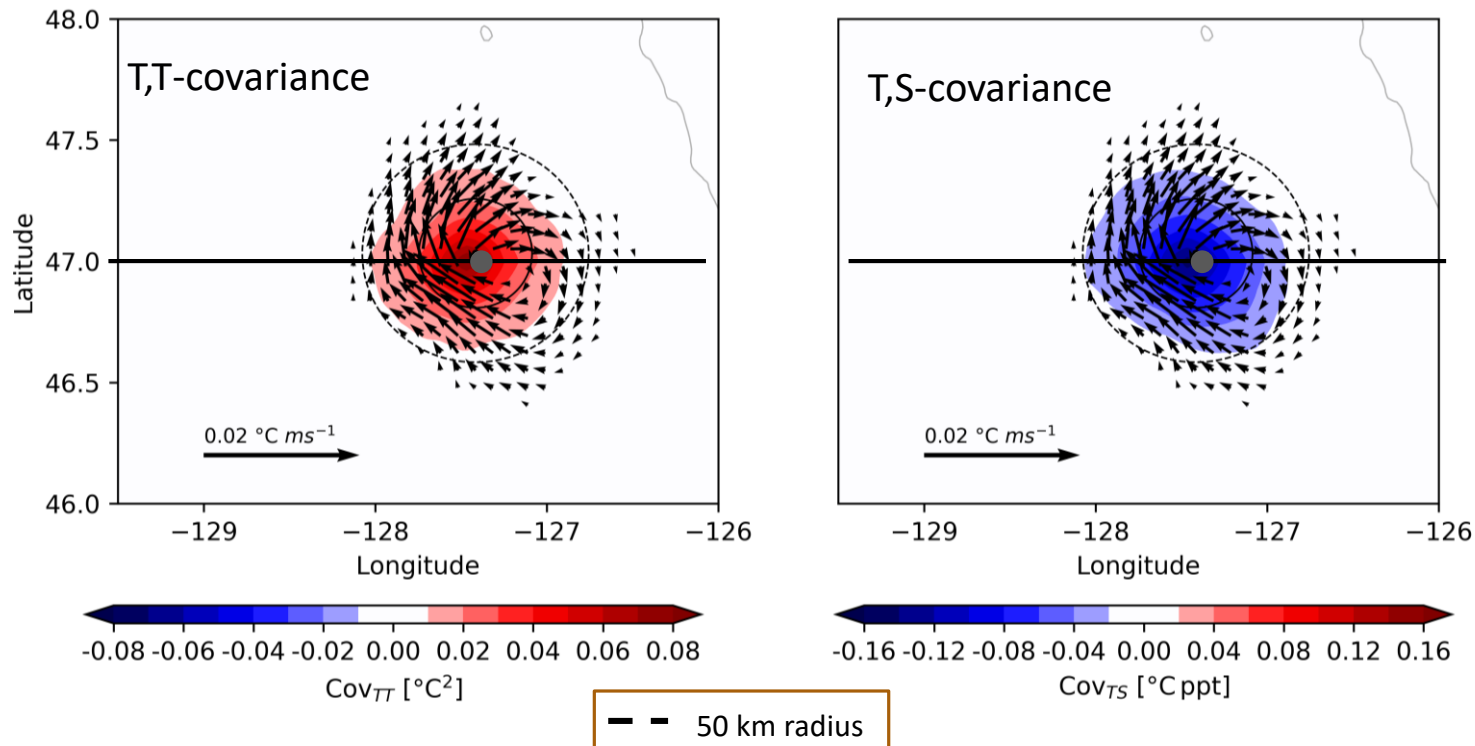
Balance operator SST, · -covariances along 47°N



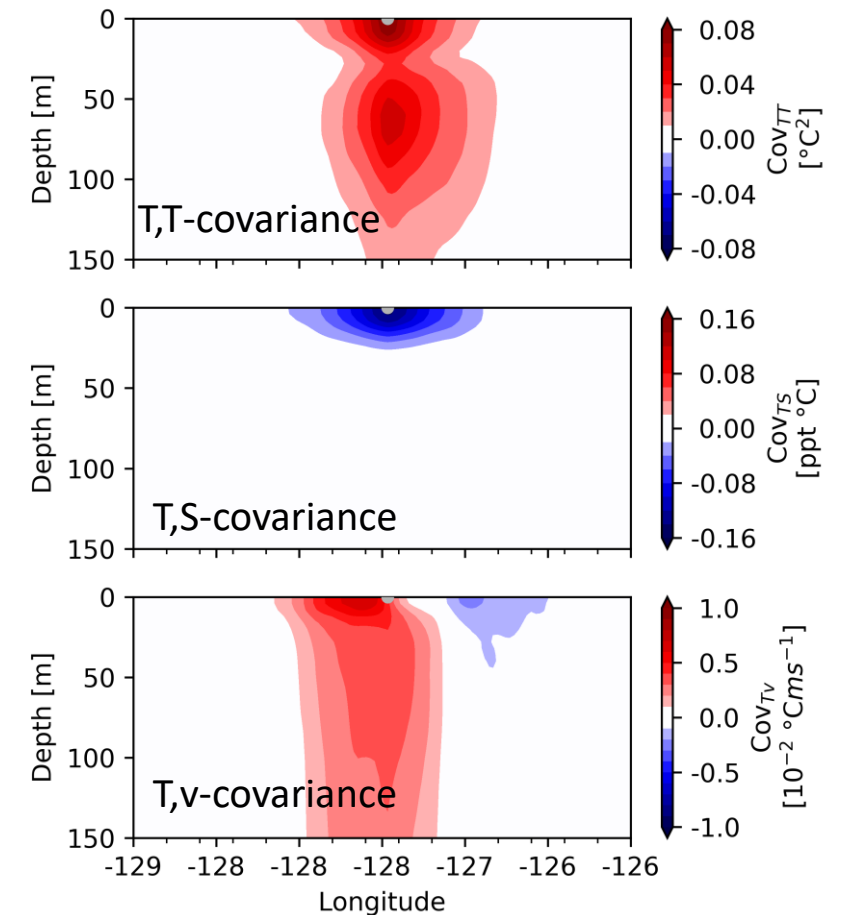
Ensemble covariance: open ocean

On average localized ensemble covariance reproduces the balance operator covariance, but with a T-variance a factor 10 smaller.

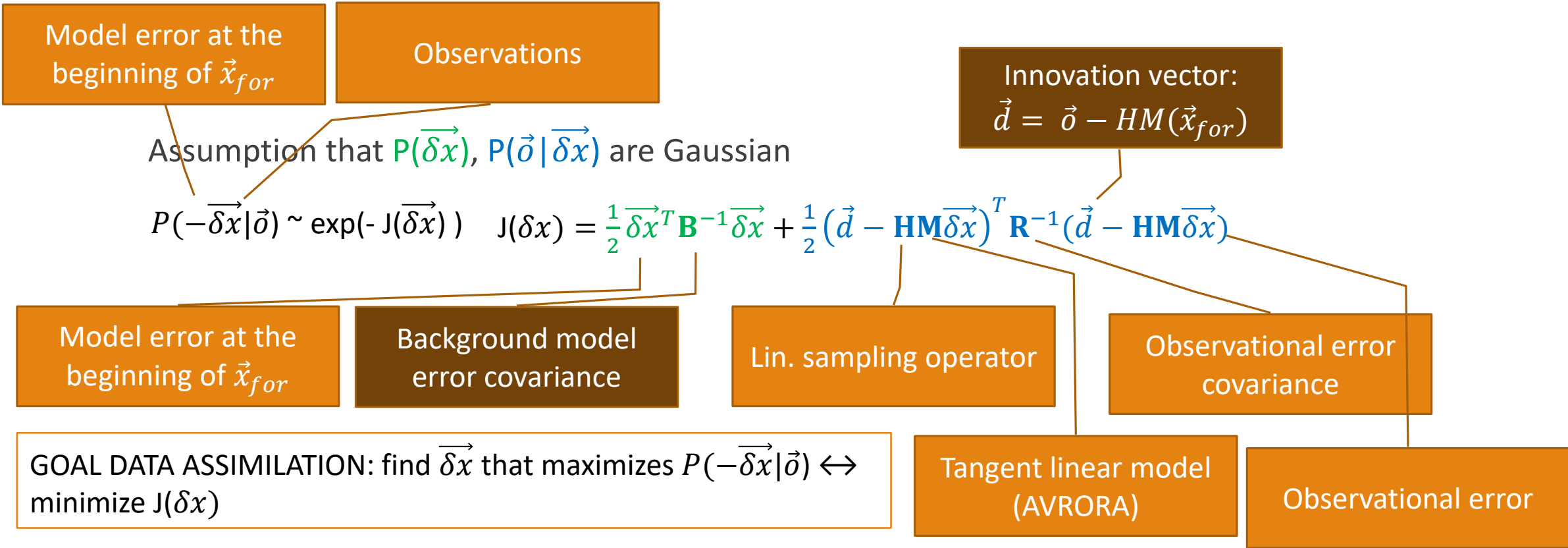
Averaged localized SST, · -covariances



Averaged SST, · -covariances along 47°N



Strong 4DVAR



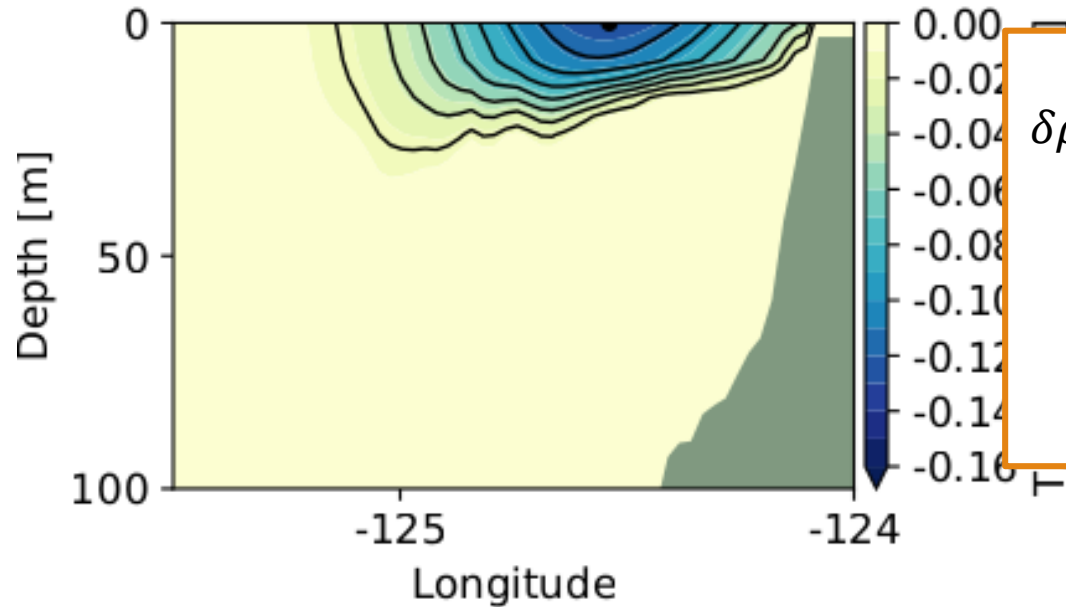
Courtier et al., 1994: "A strategy for operational implementation of 4D-Var, using an incremental approach." *Quarterly Journal of the Royal Meteorological Society*, 120, 1367-1387.

Model error covariance **B**

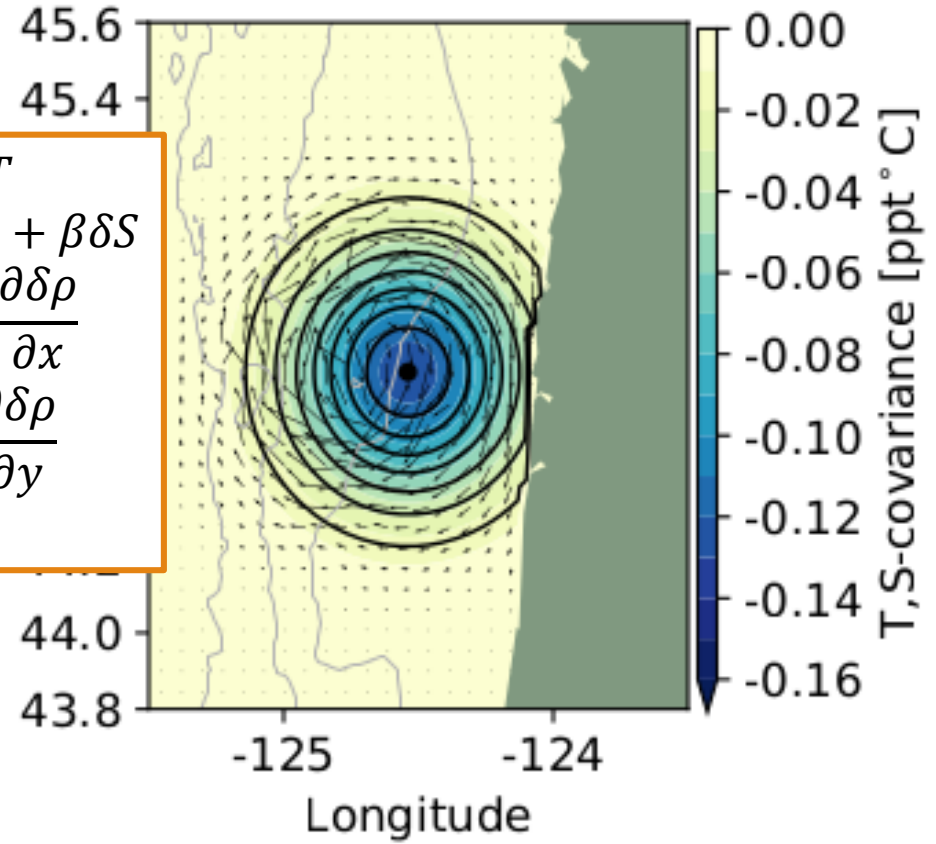
- Balance operator covariance (Weaver et al., 2005)

Temperature-salinity and temperature-velocity covariance at the surface

Temperature-salinity covariance in a zonal cross-section



$$\begin{aligned} \delta S &= \gamma \delta T \\ \delta \rho / \rho_0 &= -\alpha \delta T + \beta \delta S \\ \delta v &= -\frac{g}{\rho_0 f} \frac{\partial \delta \rho}{\partial x} \\ \delta u &= \frac{g}{\rho_0 f} \frac{\partial \delta \rho}{\partial y} \end{aligned}$$



Perturbations

- Add noise $N(0, \mathbf{R})$ to observations (Houtekamer and Mitchell, 1998)
- Wind: stochastic model based around NCEI NAM fields.

$$\vec{w}_{member}(\vec{r}, t) = \vec{w}_{NAM}(\vec{r}, t) + \vec{w}_{Large}(\vec{r}, t) + \vec{w}_{Small}(\vec{r}, t)$$

- \vec{w}_{NAM} wind from NAM model (12 km resolution)
- $\vec{w}_{Large}(\vec{r}, t)$ large scale NAM model error
- $\vec{w}_{Small}(\vec{r}, t)$ small scale NAM model error

Large-scale wind perturbations

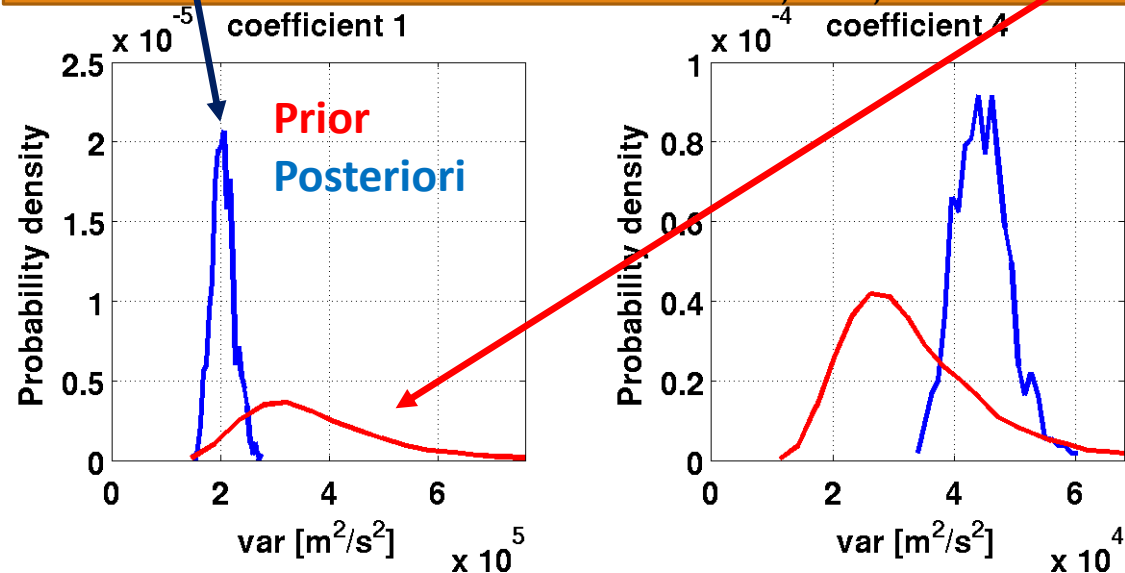
- Large scale wind perturbation:

$$\vec{w}_{Large}(\vec{r}, t) = \sum_{i=1}^{10} \lambda_i(t) EOF_i(\vec{r}) \quad [N(\vec{\lambda}; \vec{0}, \vec{\sigma}_L)]$$

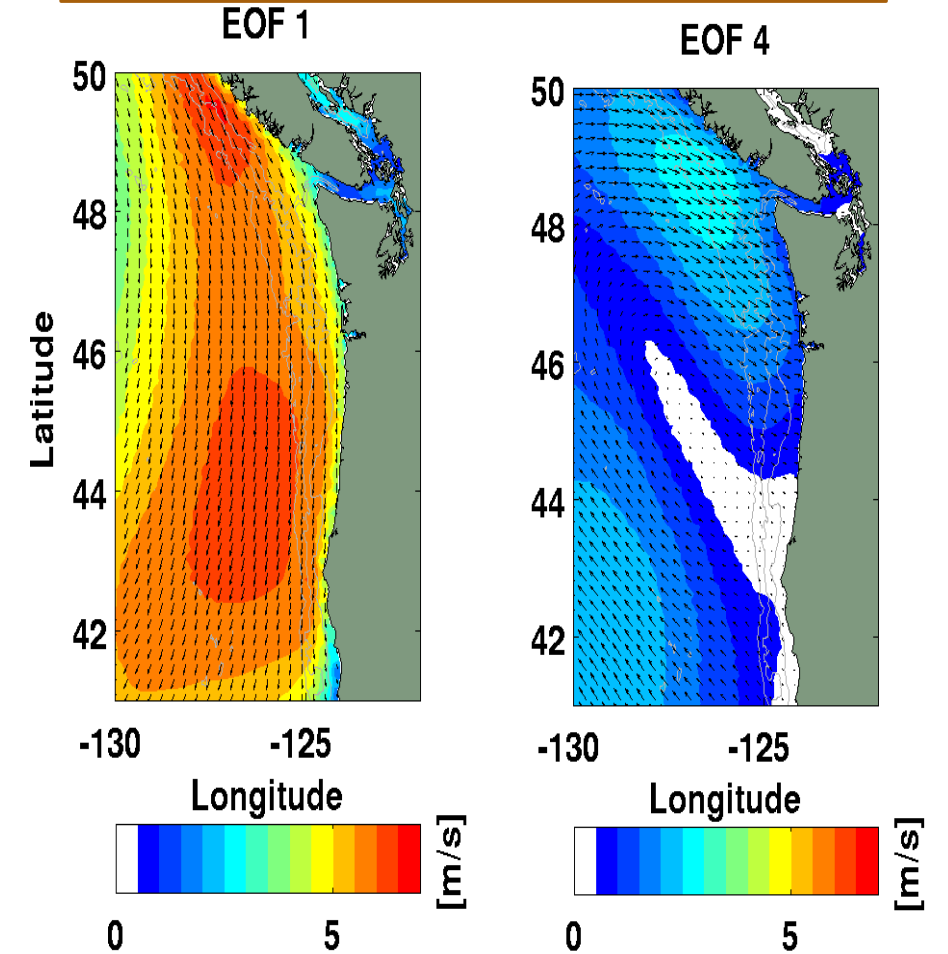
- As e.g. Hénaff et al. (2009) with modifications
- $\lambda_i(t)$ assumed to be drawn from AR1 process with zero mean. Correlation over 24h: 0.026
- Standard deviation $\lambda_i(t)$ determined by applying a Gibbs sampler to error with daily ASCAT data (Milliff et al., 2011):

$$P(\vec{\lambda}, \vec{w}_S, \vec{\sigma}_L, \sigma_S | \vec{w}_{Ascatter}) \sim N(\vec{w}_{Ascatter} | \vec{\lambda}, \vec{w}_S, \sigma_S) N(\vec{w}_S; \vec{0}, \sigma_S) IG(\sigma_S) N(\vec{\lambda}; \vec{0}, \vec{\sigma}_L) IG(\vec{\sigma}_L)$$

Probability distribution $\sigma_{L,1}, \sigma_{L,4}$



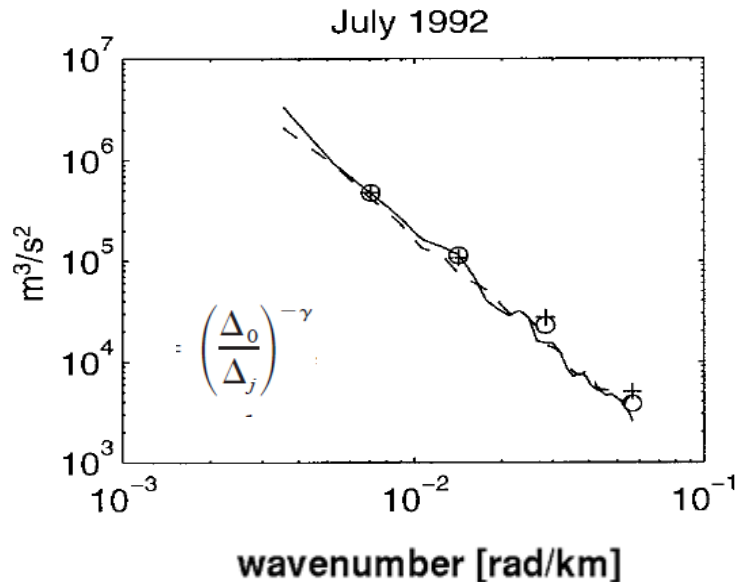
EOF wind fields



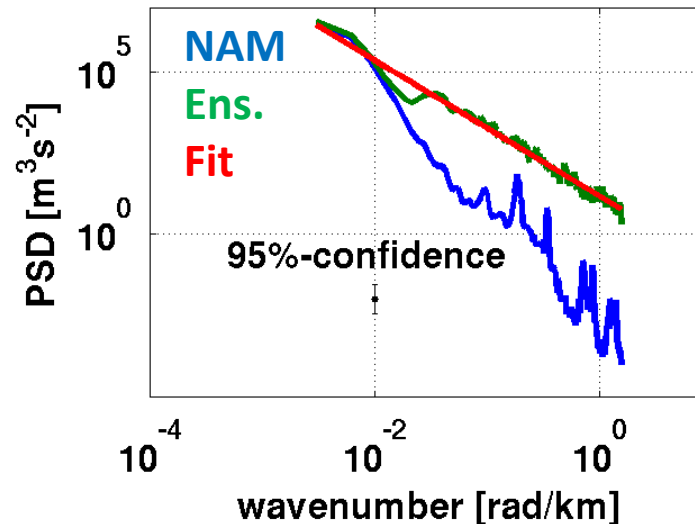
Small-scale wind perturbations

- Small scale wind perturbations:
 - Wind fields have a red spectrum
 - Linear combination of Daubechie-2 wavelets (Wikle et al.,2001) are used to reproduce this.
 - \vec{w}_S scaled such that $var(\vec{w}_S) = 2\sigma^2 = 1m^2s^{-2}$

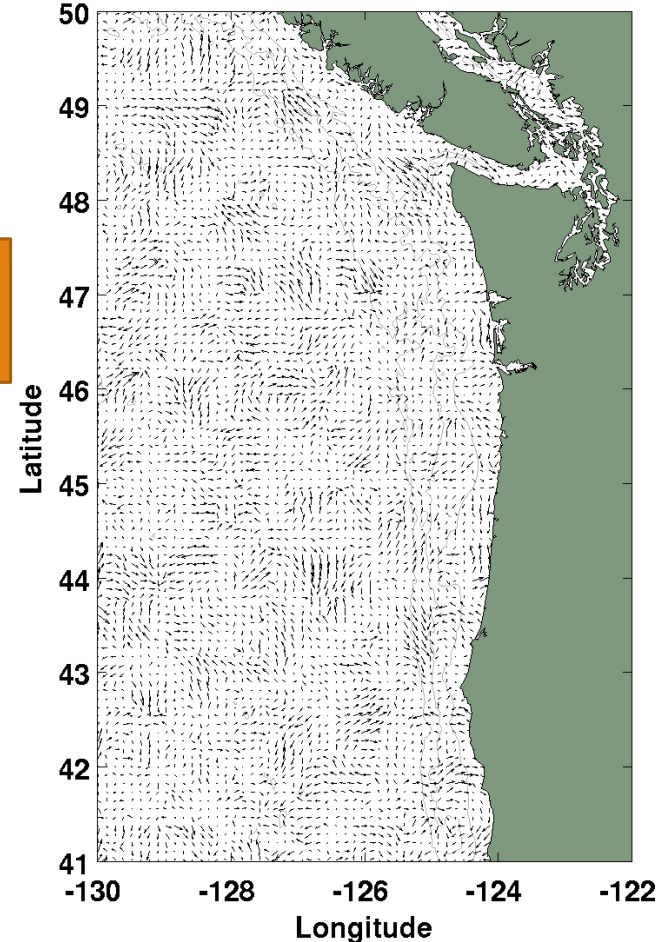
Spectral density of zonal (solid) and meridional (dashed) scatterometer winds (Chin et al., 1998)



Spectral density v in NAM and wind field ensemble members



Example small-scale wind field



Primal vs dual formulation

Goal: minimize $J(\delta x) = \frac{1}{2} \overrightarrow{\delta x}^T \mathbf{B}^{-1} \overrightarrow{\delta x} + \frac{1}{2} (\vec{d} - \mathbf{HM} \overrightarrow{\delta x})^T \mathbf{R}^{-1} (\vec{d} - \mathbf{HM} \overrightarrow{\delta x})$

■ Primal solution:

$$(\mathbf{B}^{-1} + \mathbf{M}^T \mathbf{H}^T \mathbf{R}^{-1} \mathbf{HM}) \overrightarrow{\delta x} = \mathbf{M}^T \mathbf{H}^T \mathbf{R}^{-1} \vec{d}$$

■ Dual solution:

$$(\mathbf{HMB} \quad \mathbf{M}^T \mathbf{H}^T + \mathbf{R}) \vec{\chi} = \vec{d}, \quad \overrightarrow{\delta x} = \mathbf{BM}^T \mathbf{H}^T \vec{\chi}$$

B: background error covariance
R: observational error covariance
M: tangent linear model
H: sampling operator
 \vec{d} : innovation vector
 $\overrightarrow{\delta x}$: 4DVAR correction
 $\vec{\chi}$: solution dual

Primal vs dual formulation

- Dual solution:

$$(\mathbf{HMB} \ \mathbf{M}^T \mathbf{H}^T + \mathbf{R}) \vec{\chi} = \vec{d}, \quad \overrightarrow{\delta x} = \mathbf{B} \mathbf{M}^T \mathbf{H}^T \vec{\chi}$$

- SVD preconditioned:

Calculate random representers: $\mathbf{U} \mathbf{\Lambda} \mathbf{V}^T = (\mathbf{HMB} \ \mathbf{M}^T \mathbf{H}^T + \mathbf{R}) \mathbf{\Delta}$

$$\mathbf{C}^{-1/2} = \mathbf{U} \mathbf{\Lambda}^{-1/2} \mathbf{U}^T + (\mathbf{I} - \mathbf{U} \mathbf{U}^T)$$

$$\mathbf{C}^{-1/2} (\mathbf{HMB} \ \mathbf{M}^T \mathbf{H}^T + \mathbf{R}) \mathbf{C}^{-1/2} \tilde{\chi} = \mathbf{C}^{-1/2} \vec{d}, \quad \overrightarrow{\delta x} = \mathbf{B} \mathbf{M}^T \mathbf{H}^T \tilde{\chi} = \mathbf{B} \mathbf{M}^T \mathbf{H}^T \mathbf{C}^{-1/2} \tilde{\chi}$$

- **Pro**: $\dim(\tilde{\chi}) \ll \dim(\overrightarrow{\delta x})$
- **Con**: poor convergence in primal space

B: background error covariance
R: observational error covariance
M: tangent linear model
H: sampling operator
C: preconditioner
 \vec{d} : innovation vector
 $\overrightarrow{\delta x}$: 4DVAR correction
 $\vec{\chi}$: solution dual

Problems E4DVAR

- Computationally challenging

- 4DVAR needs to be applied to multiple ensemble members.
- 4DVAR requires multiple sequential conjugate gradient iterations to solve

$$(\mathbf{R}^{-1/2} \mathbf{H} \mathbf{M} \mathbf{B} \mathbf{M}^T \mathbf{H}^T \mathbf{R}^{-1/2} + \mathbf{I}) \vec{\chi} = \mathbf{R}^{-1/2} \vec{d}, \quad \vec{\delta x} = \mathbf{B} \mathbf{M}^T \mathbf{H}^T \mathbf{R}^{-1/2} \vec{\chi}$$

$$(\tilde{\mathbf{B}} + \mathbf{I}) \vec{\chi} = \mathbf{A} \vec{\chi} = \vec{d}, \quad \vec{\delta x} = \mathbf{B} \mathbf{M}^T \mathbf{H}^T \mathbf{R}^{-1/2} \vec{\chi}$$

Projection operator

- Solution for 1:

- Conjugate gradient minimizes^{1,2} $\|\vec{\chi}_i - \vec{\chi}\|_{\tilde{\mathbf{B}}\mathbf{A}}$:

$$\vec{\chi}_i = \mathbf{V}_i (\mathbf{V}_i^T \tilde{\mathbf{B}} \mathbf{A} \mathbf{V}_i)^{-1} \mathbf{V}_i^T \tilde{\mathbf{B}} \mathbf{A} \vec{\chi} = \mathbf{V}_i (\mathbf{V}_i^T \tilde{\mathbf{B}} \mathbf{A} \mathbf{V}_i)^{-1} \mathbf{V}_i^T \tilde{\mathbf{B}} \vec{d}$$

with $\text{sp}(\mathbf{V}_i)$ being the Krylov space.

- Recycle \mathbf{V}_i : for n th ensemble member $\vec{\chi}_{n,i} = \mathbf{V}_i (\mathbf{V}_i^T \tilde{\mathbf{B}} \mathbf{A} \mathbf{V}_i)^{-1} \mathbf{V}_i^T \tilde{\mathbf{B}} (\vec{d}_n + \vec{\epsilon})$

$\vec{\epsilon}$ is drawn from standard Gaussian. Similar Krylov space reuse, but different implementation as in EVIL³.

B: background error covariance
R: observational error covariance
M: tangent linear model
H: sampling operator
 \vec{d} : innovation vector
 $\vec{\delta x}$: 4DVAR correction
 $\vec{\chi}$: solution dual

¹Trefethen, Lloyd N., and David Bau III. 1997. *Numerical Linear Algebra*. Vol. 50. Siam.

²Gurol, S., A. T. Weaver, A. M. Moore, A. Piacentini, H. G. Arango, and S. Gratton. 2014. "B-Preconditioned Minimization Algorithms for Variational Data Assimilation with the Dual Formulation." *Quarterly Journal of the Royal Meteorological Society* 140 (679): 539–56.

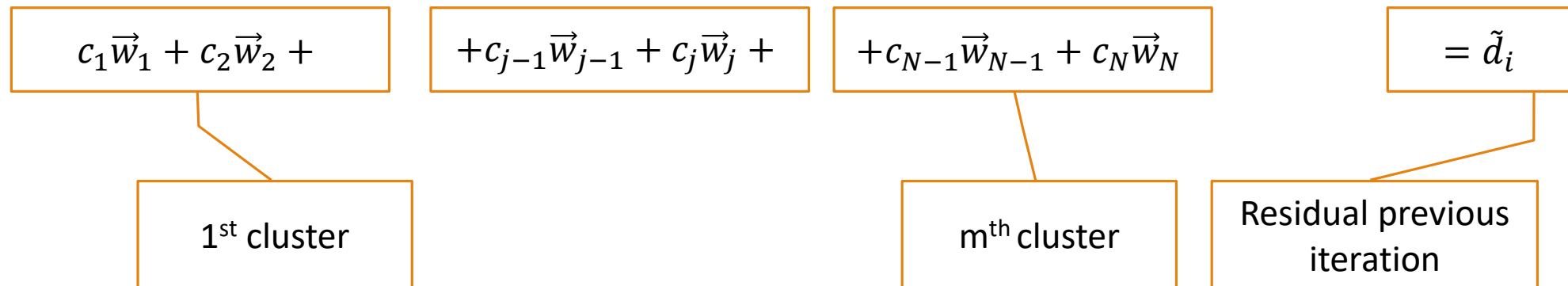
³Auligné, Thomas, Benjamin Ménétrier, Andrew C. Lorenc, and Mark Buehner. 2016. "Ensemble-Variational Integrated Localized Data Assimilation." *Monthly Weather Review* 144 (10): 3677–96

Parallel 4DVAR

Solution 2: expand $\text{sp}(\mathbf{V}_i)$ with $m > 1$ vectors per iteration.

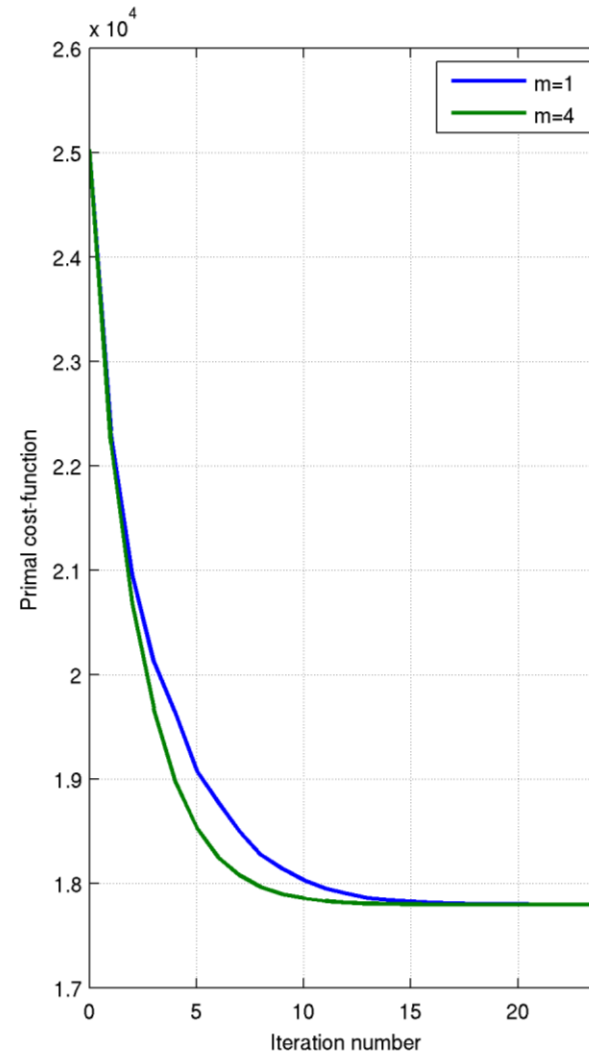
How to find these vectors:

- Use $\langle \tilde{d}\tilde{d}^T \rangle = (\tilde{\mathbf{B}} + \mathbf{I})$ and SVD to estimate eigenvalues and eigenvector of $\tilde{\mathbf{B}} \approx \mathbf{W}\mathbf{\Lambda}\mathbf{W}^T$
- Search for linear combinations eigenvectors.
- Minimization error $\|\chi - \chi_{i+1}\|_{\tilde{\mathbf{B}}\mathbf{A}} \leftrightarrow$ minimization weighted in cluster variance \rightarrow K-means



Problems E4DVAR

- Parallel method makes minimization with less than 13 iterations feasible.



Ensemble covariance localization

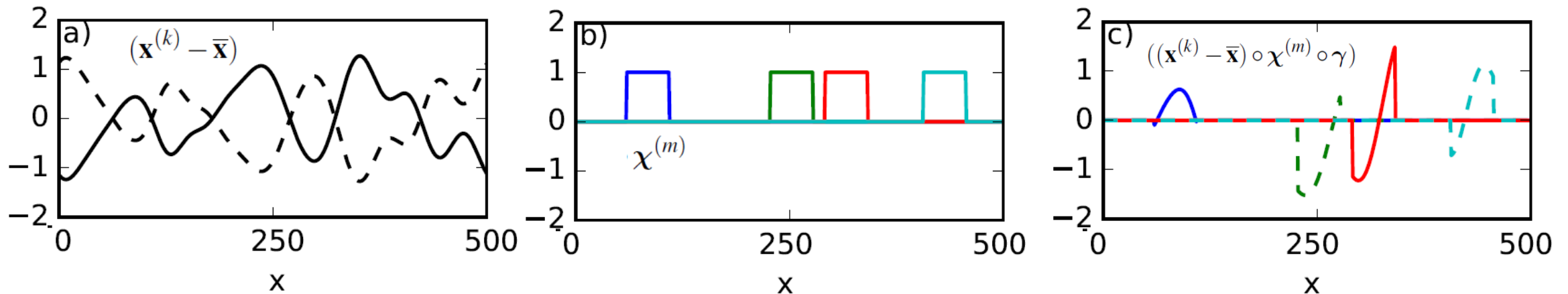
- Raw ensemble covariance:
$$\mathbf{B}_{ens} = \frac{1}{K-1} \sum_{k=1}^K (\mathbf{x}^{(k)} - \bar{\mathbf{x}})(\mathbf{x}^{(k)} - \bar{\mathbf{x}})^T$$

B: background covariance
x: ensemble member
 $\bar{\mathbf{x}}$: ensemble mean

with \mathbf{x} the daily-averaged fields at the beginning of the assimilation window

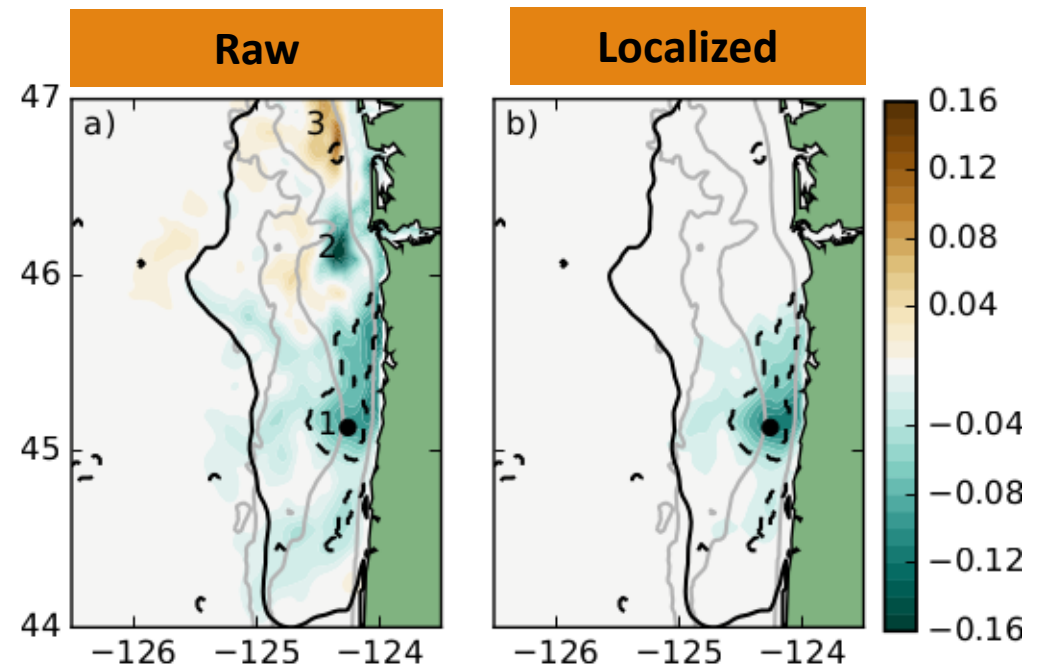
- Localized ensemble covariance using MC localization (Pasmans and Kurapov, MWR 2017 in press)

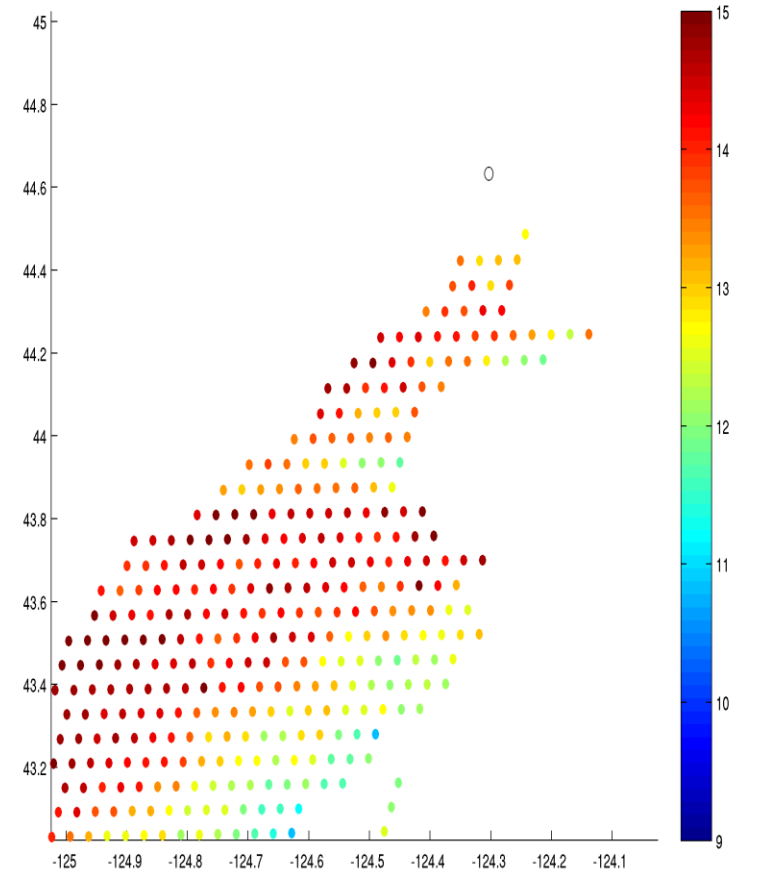
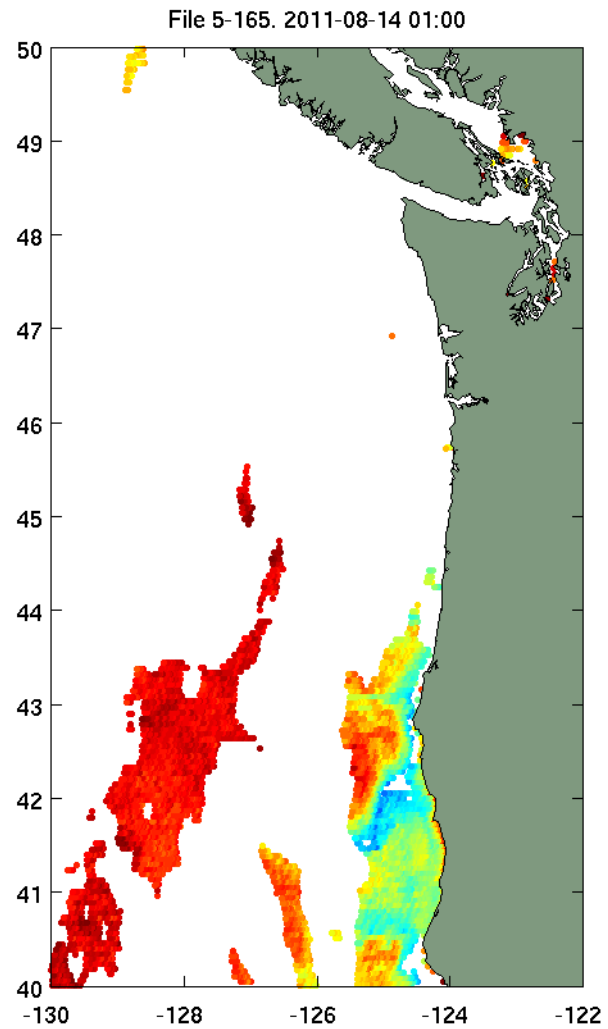
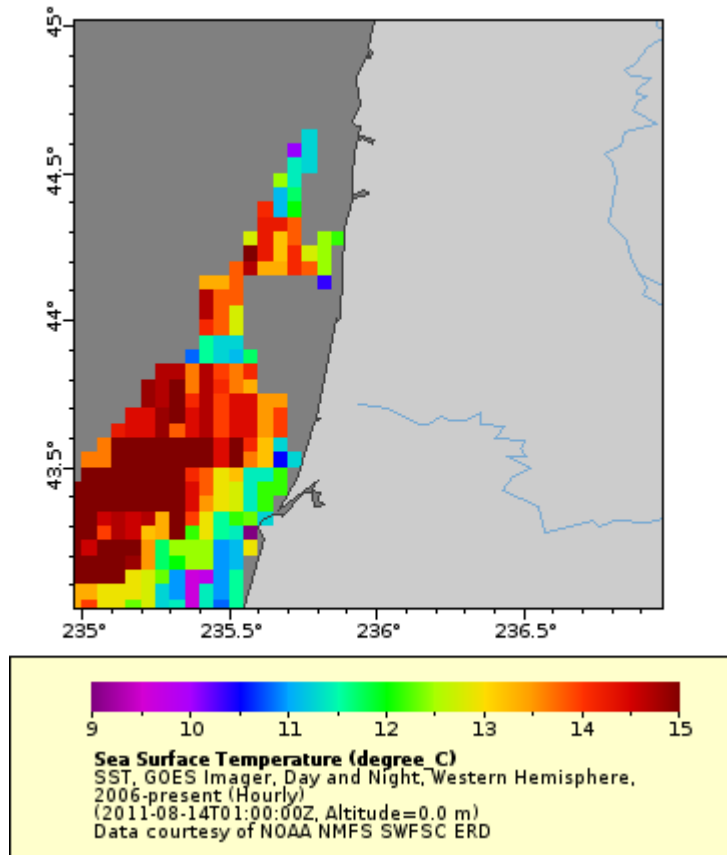
$$\mathbf{B}_{MC} = \frac{1}{K-1} \sum_{k=1}^K \sum_{m \in \mathbb{M}_k} ((\mathbf{x}^{(k)} - \bar{\mathbf{x}}) \circ \chi^{(m)} \circ \gamma) ((\mathbf{x}^{(k)} - \bar{\mathbf{x}}) \circ \chi^{(m)} \circ \gamma)^T, \quad \gamma_p = \left(\frac{1}{K} \sum_{k=1}^K \sum_{m \in \mathbb{M}_k} (\chi_p^{(m)})^2 \right)^{-1/2}$$

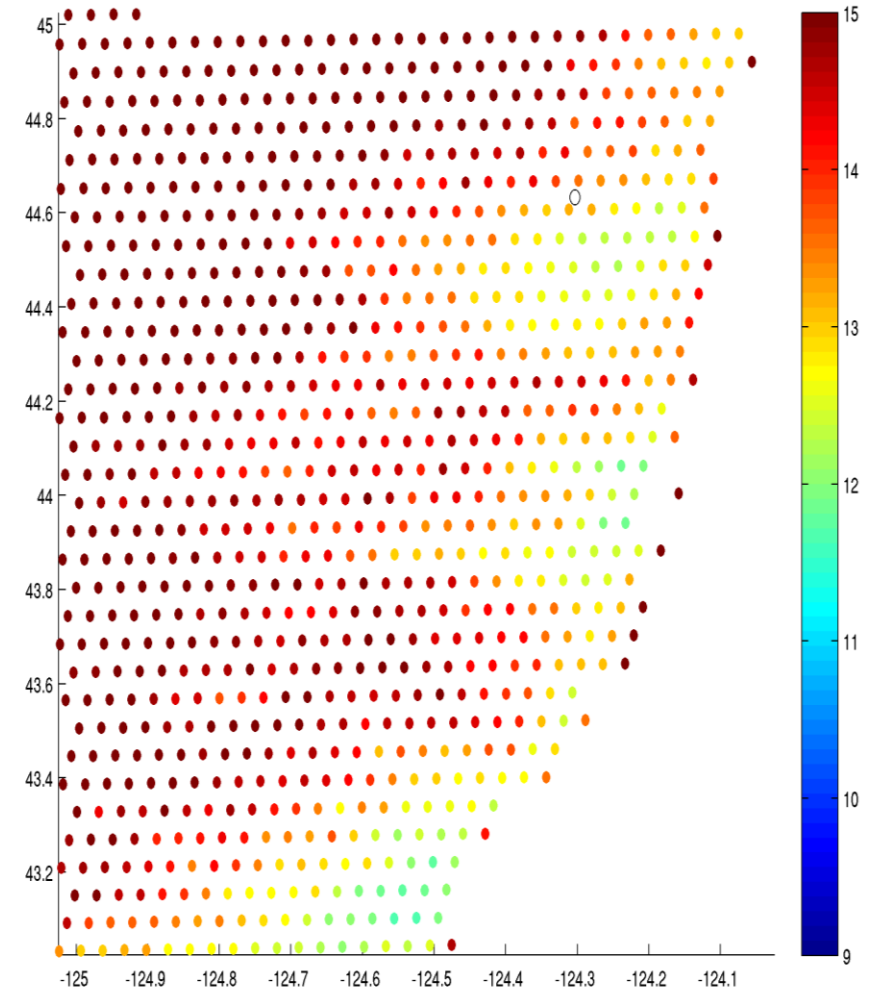
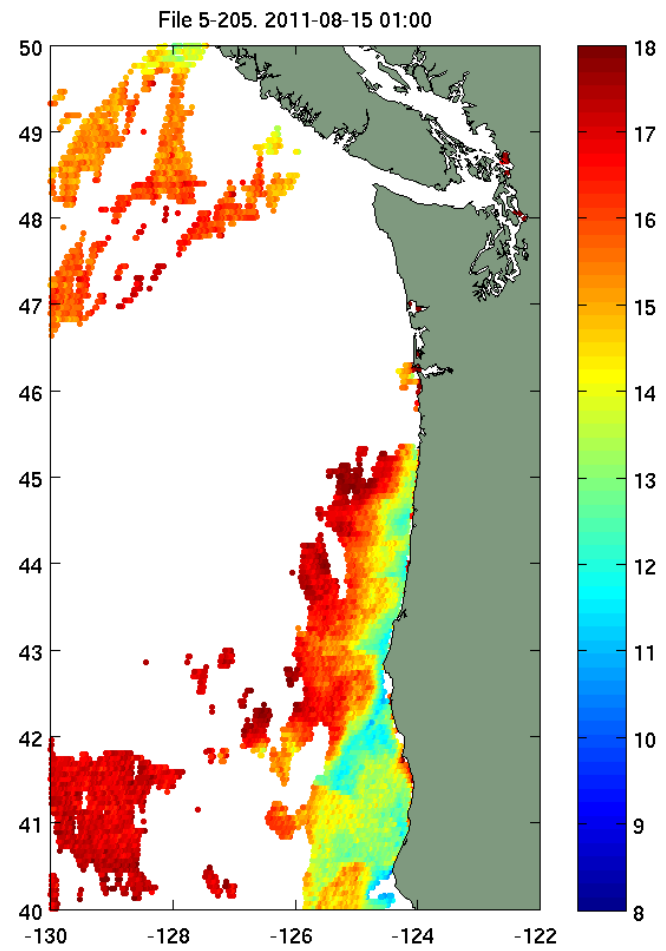
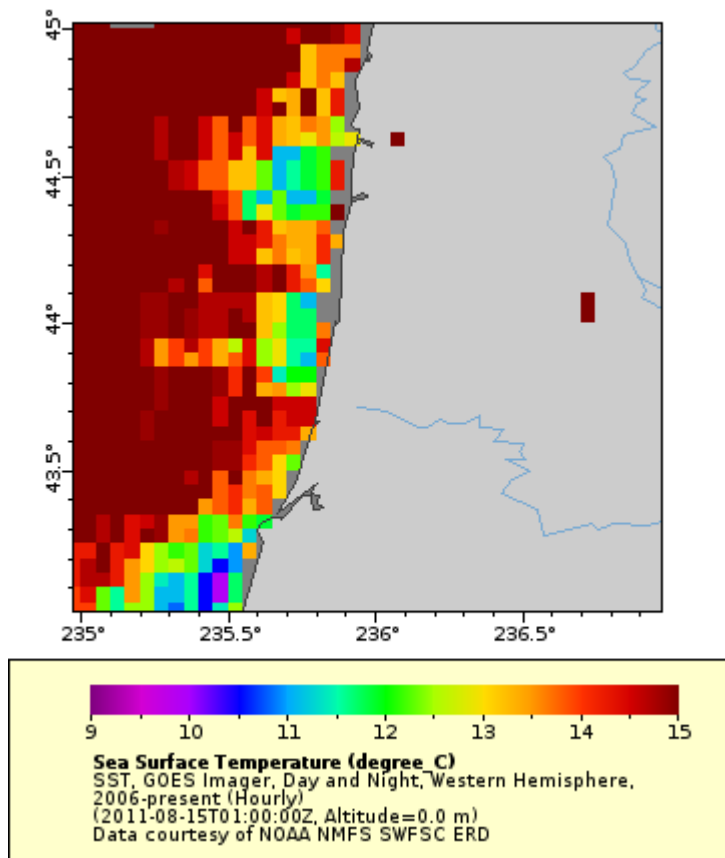


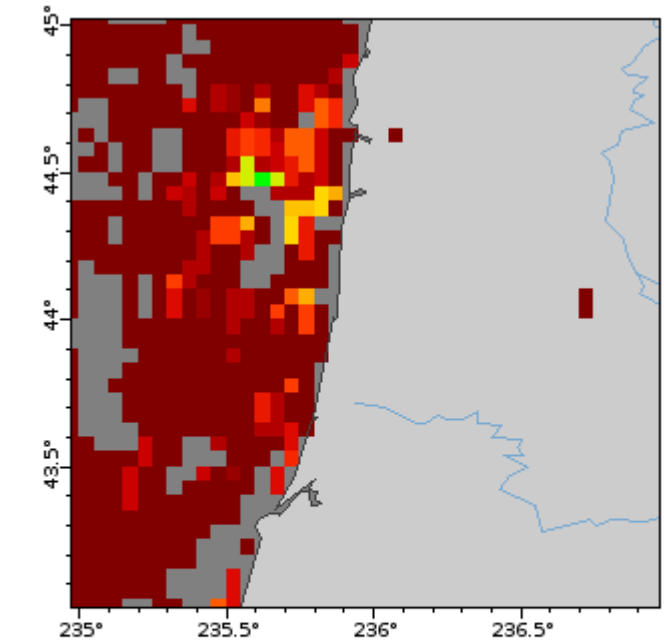
Ensemble covariance localization

- Faster computation than Gaspari and Cohn (1999) if localization distances are comparable and number of masks $<$ number of grid points
- Smooths spectrum ensemble members



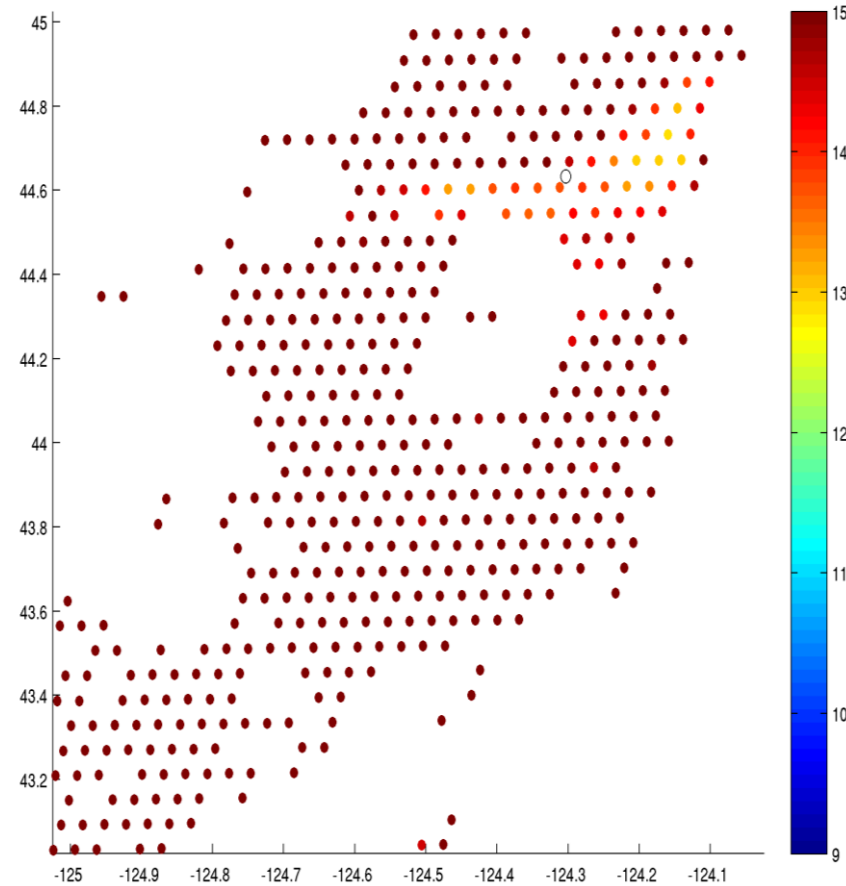
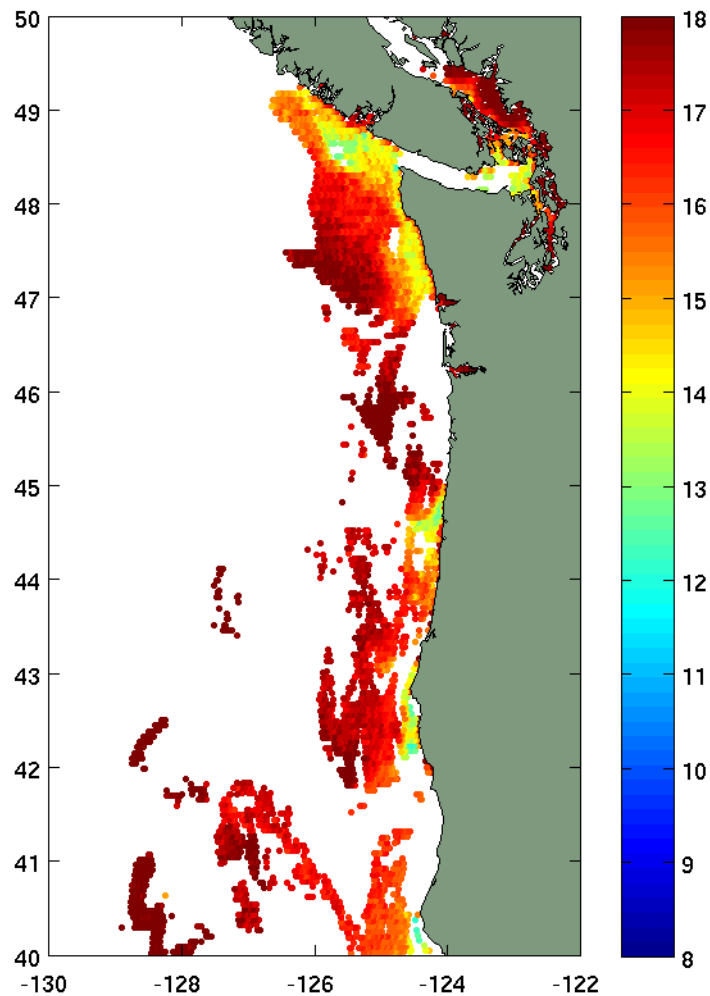






Sea Surface Temperature (degree C)
 SST, GOES Imager, Day and Night, Western Hemisphere,
 2006-present (Hourly)
 (2011-08-24T02:00:00Z, Altitude=0.0 m)
 Data courtesy of NOAA NMFS SWFSC ERD

File 5-530. 2011-08-24 02:00



Motivation

Forecasting of ocean conditions serves economical, safety and environmental purposes.

Fishing (photo source: amigocharters.com)



Debris tracking



Oil spill modeling (image source: Greenpeace)

