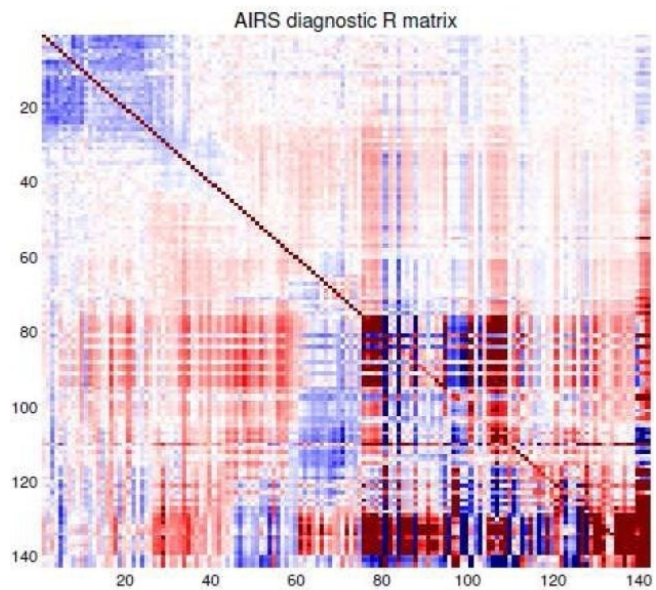
Incorporating Correlated Observation Errors in Variational Data Assimilation



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** Met Office

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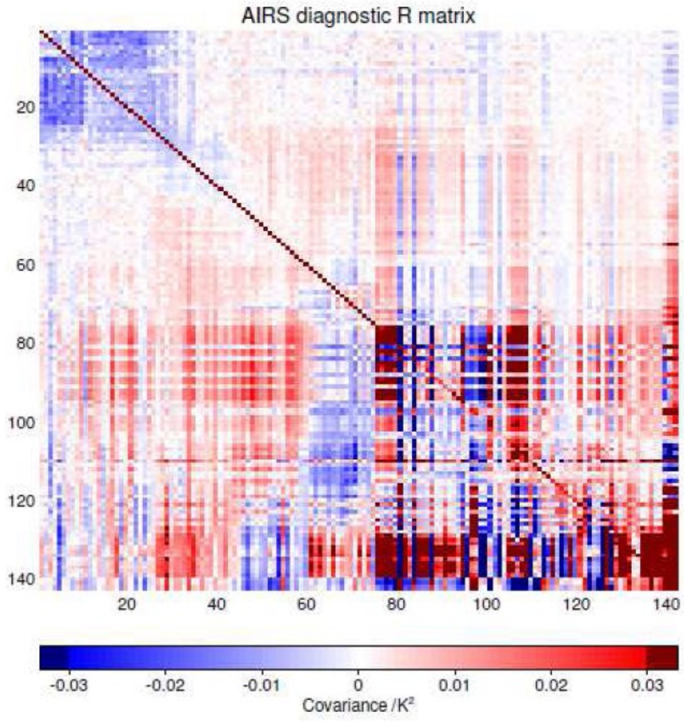
Observation Errors



Observation Error Covariance Matrix

- Observation errors assumed uncorrelated in data assimilation

- Observation errors in real data are found to be correlated
(*Stewart et al, 2009, 2013; Bormann et al, 2010; Waller et al, 2013, 2014a*.)

- Using observation error correlations in data assimilation is shown to improve the analysis
(*Stewart et al, 2008, 2010, 2014; Weston, 2014*.)

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Observation Errors

It is important to be able to account for observation error correlations:

- More of available data used (avoids thinning)
- More information content
- Better analysis accuracy
- Improved NWP forecast skill scores

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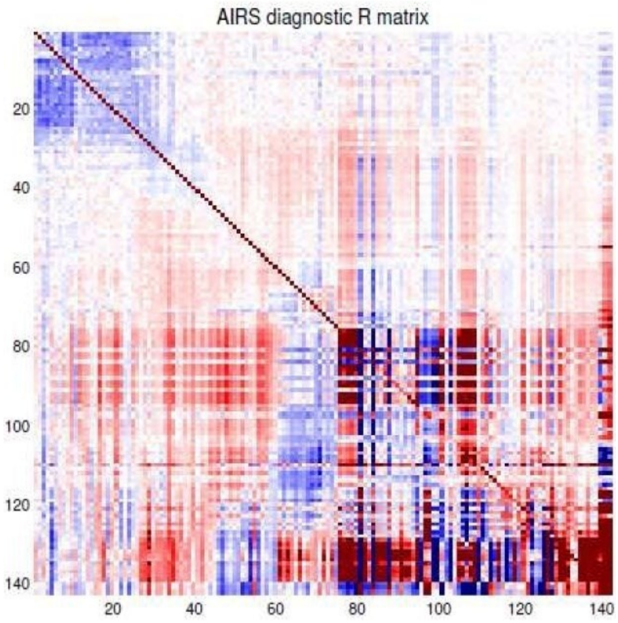
Error correlations can be diagnosed by techniques such as Deroziers et al (DBCP) or Hollingsworth-Lönnberg

Problems for DA:



Diagnosed correlation matrices:

- Non-symmetric
- Variances too small
- Not positive-definite
- Very ill-conditioned

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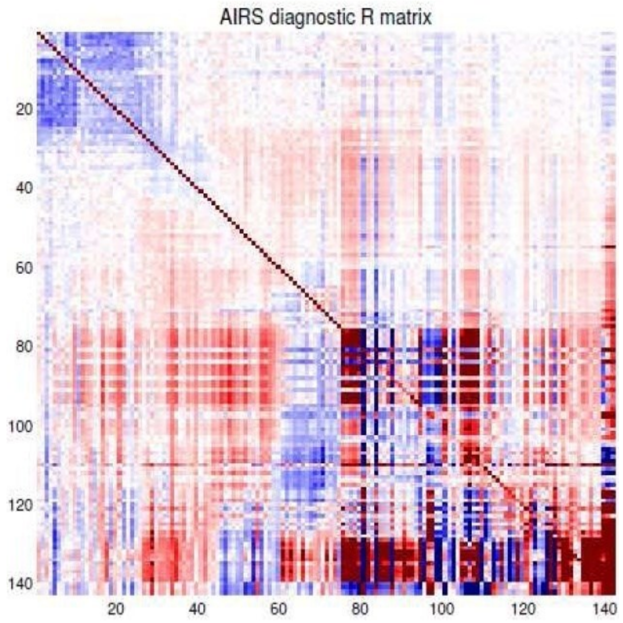

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Result: including observation error correlations in 4DVar slows convergence catastrophically

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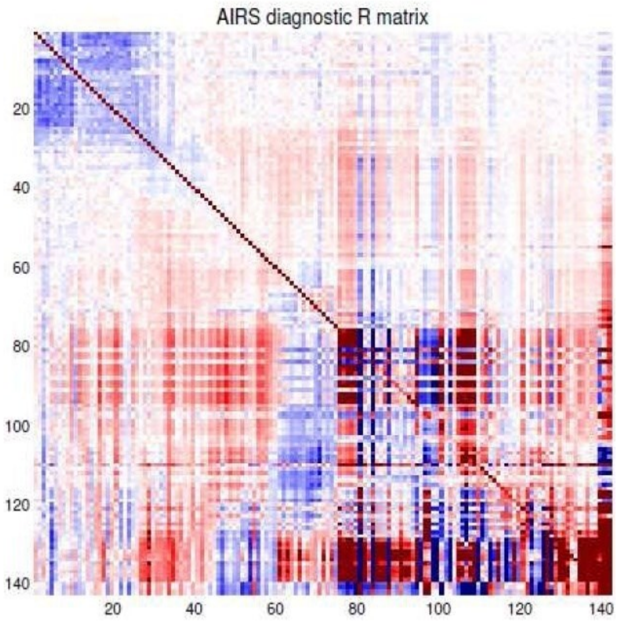

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Aim: to understand / enable the use of (diagnosed) correlated observation errors in DA

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Optimal Bayesian Estimate

Minimize with respect to initial state $\mathbf{x}_0$:

$$J(\mathbf{x}_0) = \frac{1}{2}(\mathbf{x}_0 - \mathbf{x}_0^b)^T \mathbf{B}^{-1}(\mathbf{x}_0 - \mathbf{x}_0^b) + \frac{1}{2}(\mathcal{H}(\mathbf{x}_0) - \mathbf{y})^T \mathbf{R}^{-1}(\mathcal{H}(\mathbf{x}_0) - \mathbf{y})$$

- Background state $\mathbf{x}_0^b$
- Observations $\mathbf{y}$
- Observation operator $\mathcal{H}$
- Error covariance matrices $\mathbf{B}$, $\mathbf{R}$

The solution at the minimum, x^a , is the analysis.

Sensitivity of the Problem

Rate of convergence and accuracy of the solution are bounded in terms of the condition number of the Hessian of the variational cost function:

$$\kappa(\mathbf{S}) = \lambda_{\text{max}}(\mathbf{S})/\lambda_{\text{min}}(\mathbf{S})$$

where λ denotes an eigenvalue and the Hessian is:

$$\mathbf{S} = \mathbf{B}^{-1} + (\mathbf{H})^{T}\mathbf{R}^{-1}\mathbf{H}$$

Conditioning of Hessian

We can establish the following theorem:

Let $\mathbf{B} \in \mathbb{R}^{N\times N}$ *and* $\mathbf{R} \in \mathbb{R}^{p\times p}$, *with* $p < N$, *be the background and observation error covariance matrices respectively. Additionally, let* $\mathbf{H} \in \mathbb{R}^{p\times N}$ *be the observation operator. Then the following bounds are satisfied by the condition number of the Hessian,* $\mathbf{S} = \mathbf{B}^{-1} + \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}$,

$$\frac{\kappa(\mathbf{B})}{\left(1 + \frac{\lambda_{max}(\mathbf{B})}{\lambda_{min}(\mathbf{R})}\lambda_{max}(\mathbf{H}\mathbf{H}^T)\right)} \leq \kappa(\mathbf{S}) \leq \left(1 + \frac{\lambda_{min}(\mathbf{B})}{\lambda_{min}(\mathbf{R})}\lambda_{max}(\mathbf{H}\mathbf{H}^T)\right)\kappa(\mathbf{B}).$$

Haben et al, 2011; Haben 2011, Tabeart, 2016, Tabeart et al,2018

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Now the upper bound grows as $\frac{1}{\lambda_{min}(\mathbf{R})}$ grows and depends also on the observation operator.

Haben et al, 2011; Haben 2011, Tabeart, 2016, Tabeart et al,2018

Summary: Conditioning of the Problem

We find that the condition number of S increases as:

- the observations become more accurate
- the observation spacing decreases
- the prior (background) becomes less accurate
- the prior error correlation length scales increase
- the observation error covariance becomes ill-conditioned

Haben et al, 2011; Haben 2011, Tabeart, 2016, Tabeart et al,2018

Reconditioning R

To improve the conditioning of R (and S) we alter the eigenstructure of R so as to obtain a specified condition number for the modified covariance matrix by:

- Ridge regression - add constant to all diagonal elements.
- Eigenvalue modification: increase the smallest eigenvalues of R to a threshold value that ensures the desired condition number, keeping the rest unchanged.

Details given in talk by Jemima Tabeart.

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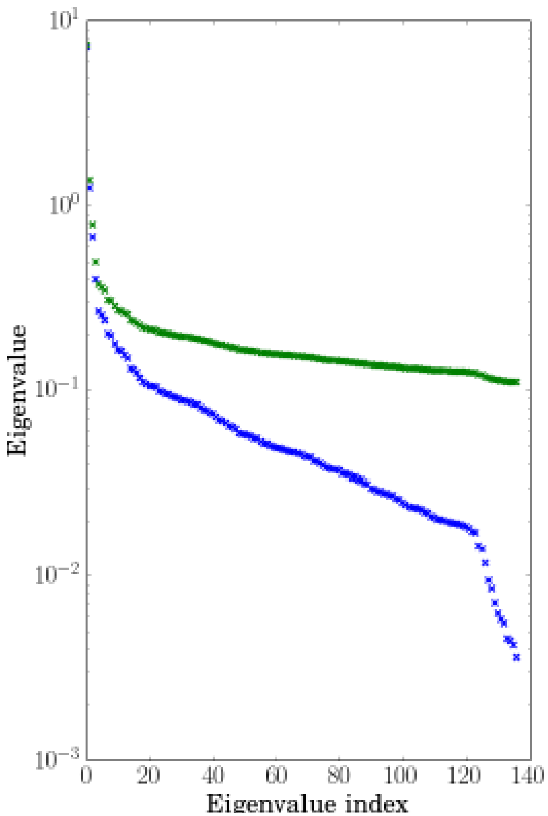
Operational Tests - Met Office

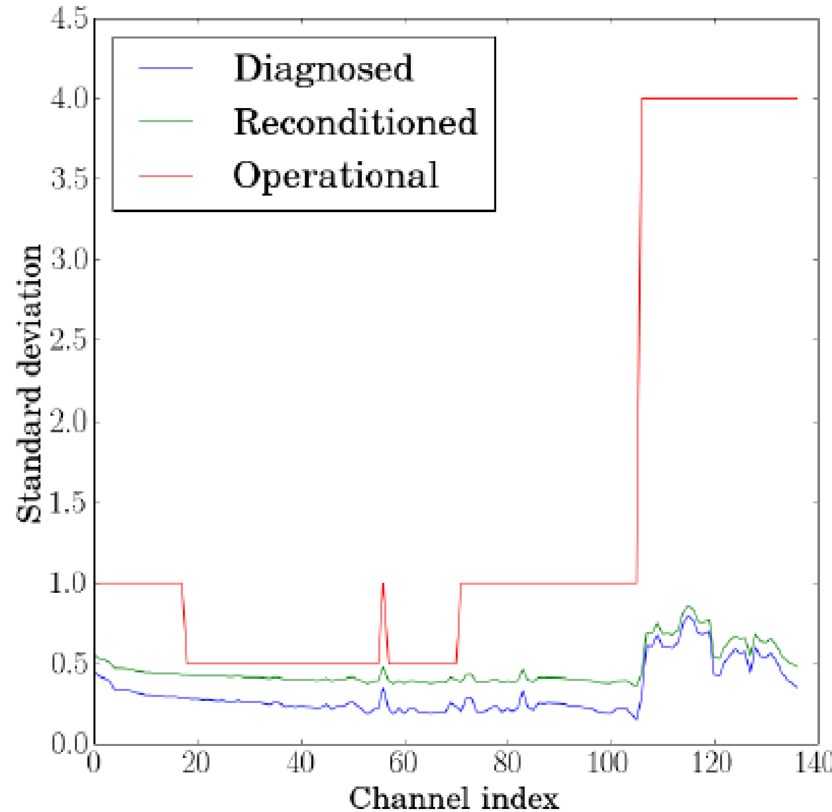
Experiments using the Met Office 1-DVar Observation Pre-processing System (OPS) for retrievals:

- Aim to test qualitative conclusions in an operational system.
- Focus on observations from IASI (Infrared Atmospheric Sounding Interferometer) instrument (on MetOp-A satellite). Note the observation operator is non-linear in this case.
- Investigate how changing the minimum eigenvalue of R affects the convergence of the iterations – we only show results using the ridge regression method.

Results - 1



(a) Change in eigenvalues of $\mathbf{R}^{137}$



(b) Standard deviations for different $\mathbf{R}^{137}$

The method of reconditioning is that described in Algorithm 1. Figure (a) depicts how the ridge regression method changes the eigenvalues of the covariance matrix $\mathbf{R}^{137}$. Eigenvalues of the raw Desroziers $\mathbf{R}^{137}_{unpre}$ are shown in blue, and those of $\mathbf{R}^{137}_{RC}$ reconditioned so that $\kappa(\mathbf{R}_{RC}) = 67$ are shown in green. Figure (b) compares the standard deviations of $\mathbf{R}^{137}_{old}$ (red), $\mathbf{R}^{137}_{unpre}$ (blue), and $\mathbf{R}^{137}_{67}$ (green).

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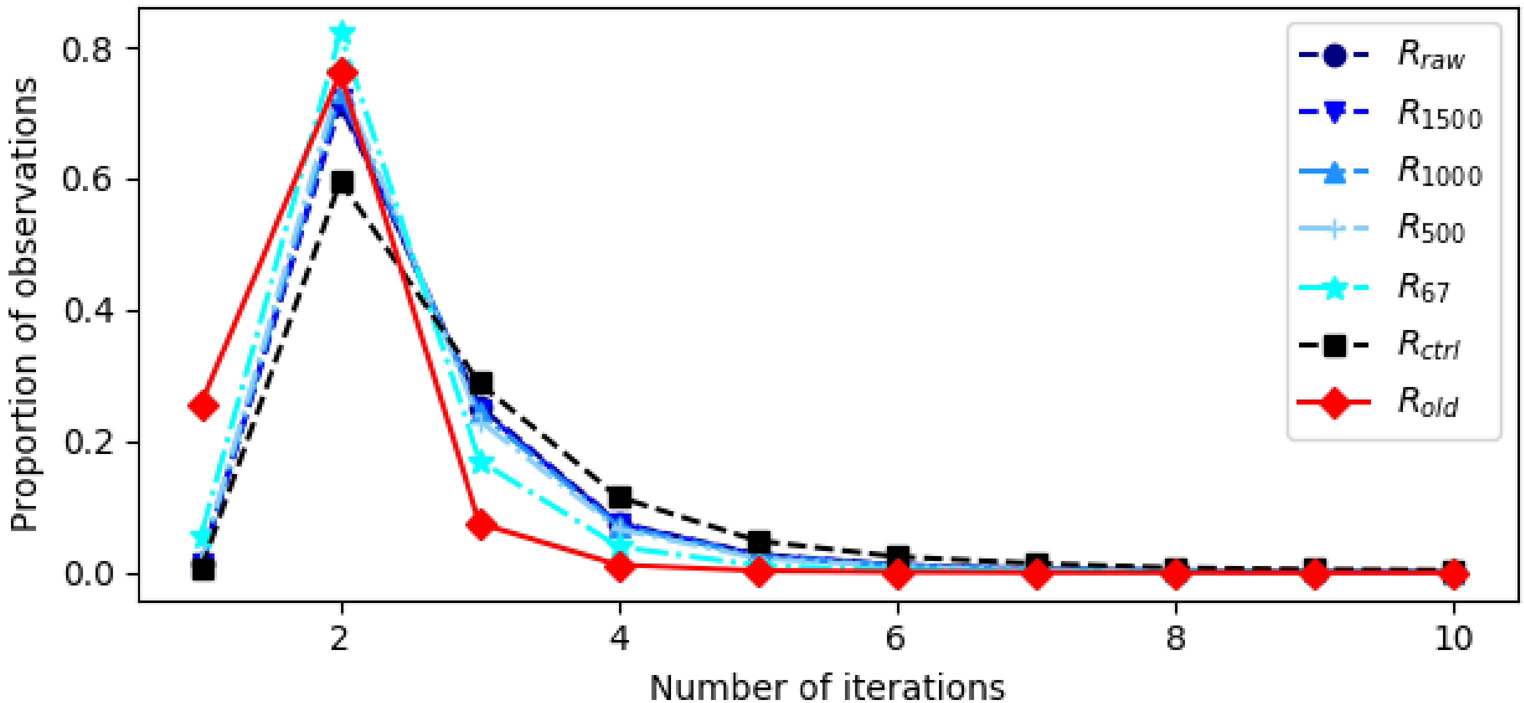
Results - 2



R$_{raw}$ – Raw (symmetrised) matrix; **R**$_{old}$ – Old MO diagonal matrix;
R$_{ctrl}$ – Current MO diagonal; **R**$_{1500}$ – Reconditioned with κ = 1500;
R$_{1000}$ – Reconditioned κ = 1000; **R**$_{500}$ – Reconditioned with κ = 500;
R$_{67}$ – Reconditioned with κ = 67;

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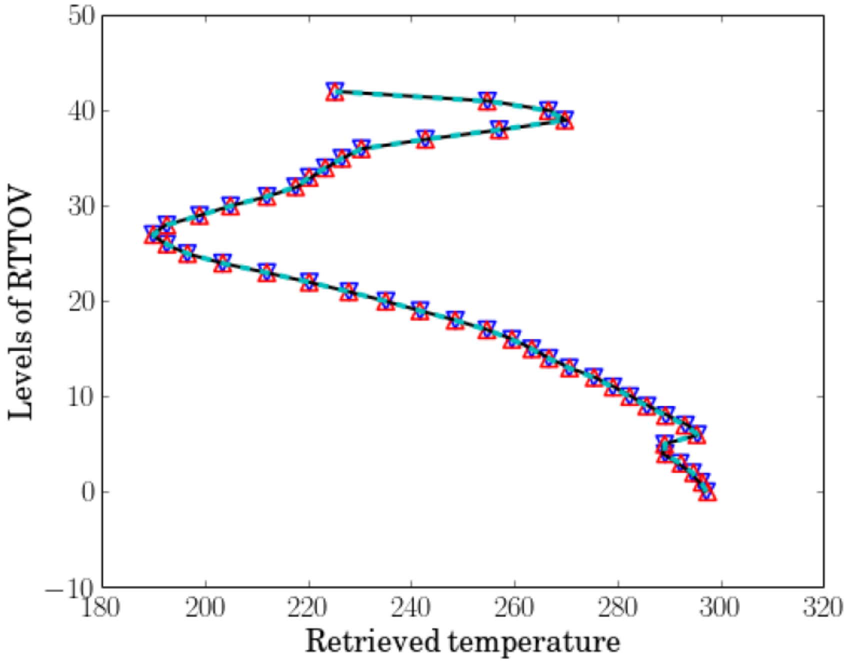
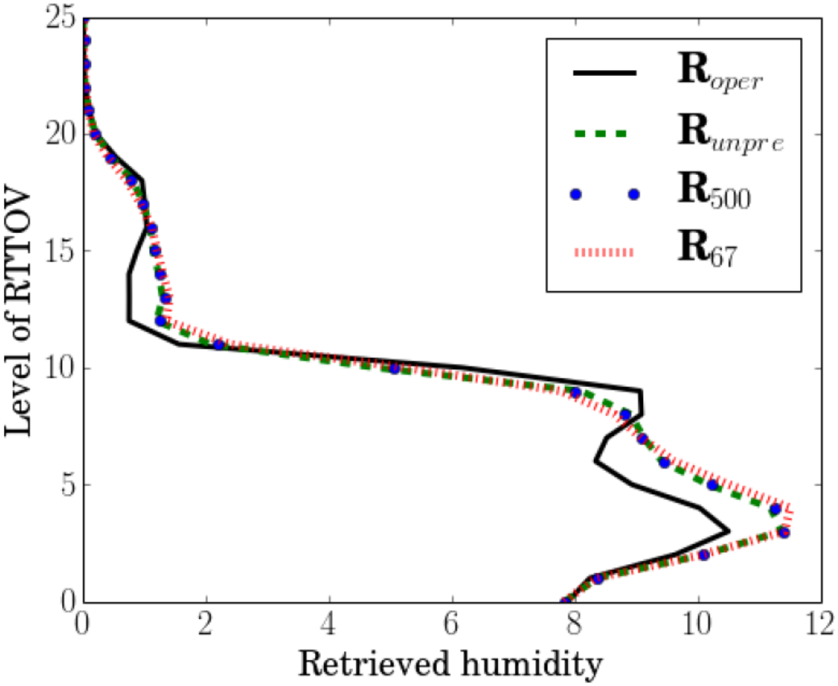
Results - 3





Shown are the retrieved temperature and humidity profiles for 4 different choices of R: R_{oper}, R_{unpre}, R500 and R67.

Summary: Operational Experiments

- Investigated the effects including observation error correlations in Met Office 1-D Var system.
- Impact on temperature retrievals was minimal, the impact on humidity retrievals much larger.
- Reducing the condition number of R reduces the number of iterations required for convergence.
- Decreasing the observation error variance increases the required number of iterations.

Tabeart, 2016; Tabeart et al, in prep

Future

Many more challenges left!

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