Uncertainty Quantification as a Tool for Observing System Design

An Oceanographic Perspective

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Patrick Heimbach (UT Austin)
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Adjoint Workshop, Aveiro, July 2018

Observing systems in the Subpolar North Atlantic

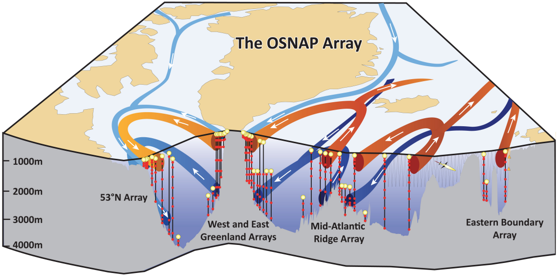
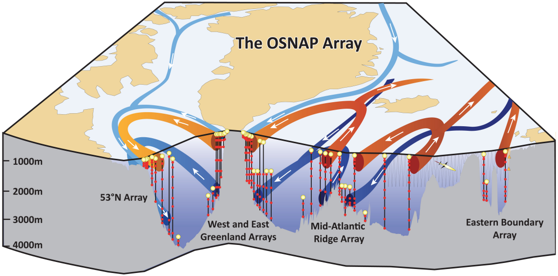


Figure: OSNAP array of the Overturning in the Subpolar North Atlantic Program (http://www.o-snap.org)

OSNAP array:

- Deployed in summer 2014

Observing systems in the Subpolar North Atlantic



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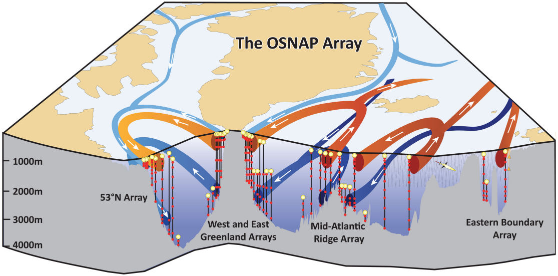

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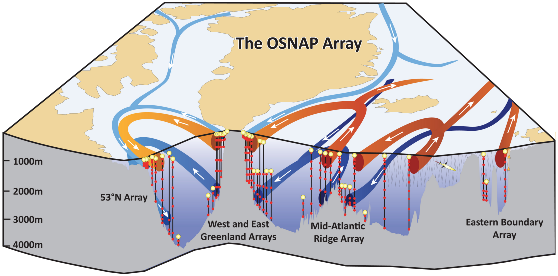

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OSNAP array:

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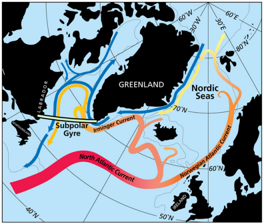
- Design based on dynamical considerations
- Review in 2020
- Data redundancy?

Adjoint sensitivities of OSNAP heat transport



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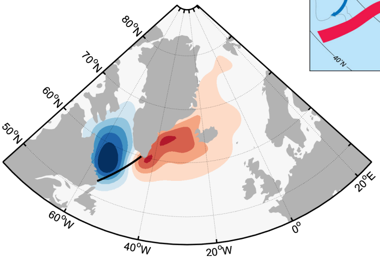

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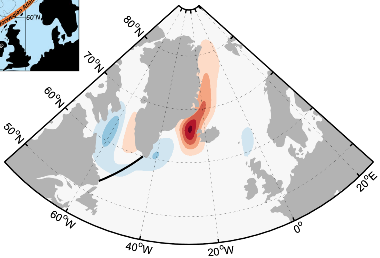

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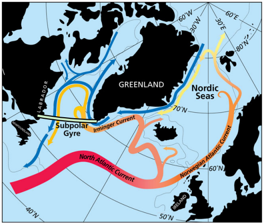

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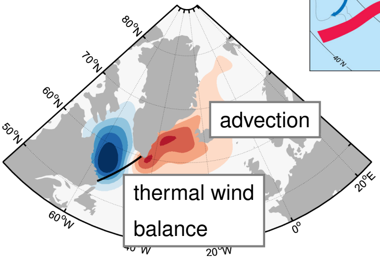

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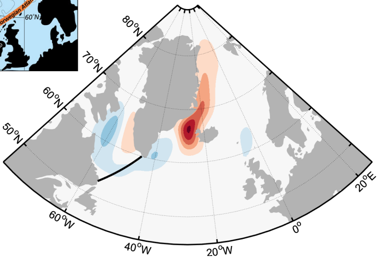

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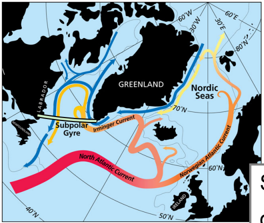

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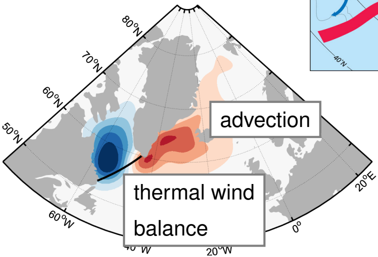

advection

thermal wind balance

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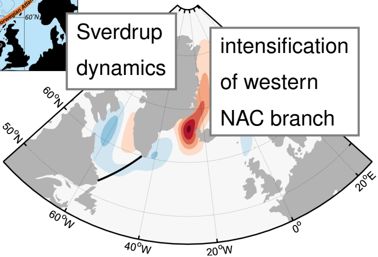

Sverdrup dynamics

intensification of western NAC branch

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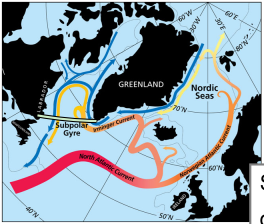

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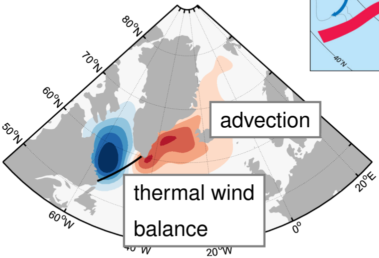

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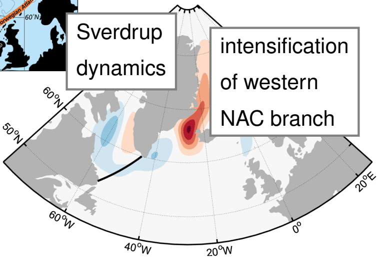

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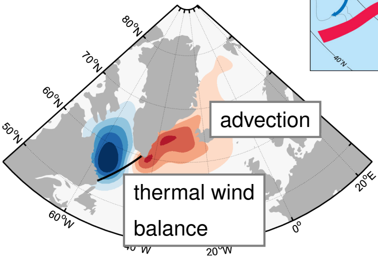

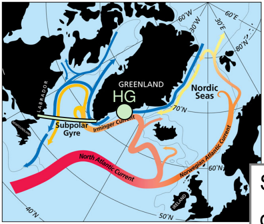

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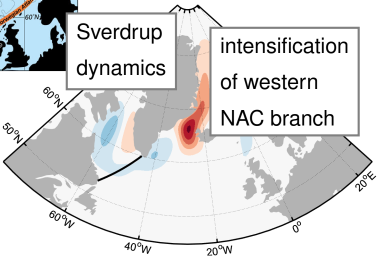

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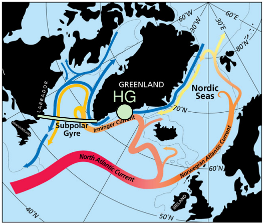

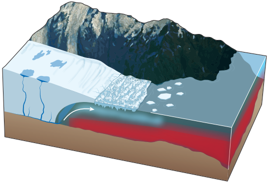

Figure: Submarine melt in Greenland's glacial fjords [Straneo and Heimbach, 2013]

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- Potential for constraining remote quantities of interest (QoIs)?
  - e.g., QoI = subsurface temperature near Helheim Glacier

Framework and Objectives

Inverse Modeling Framework (4D-Var): ECCO-V4 [Forget et al., 2015]

Final Goal:

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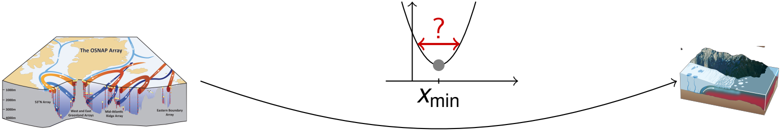

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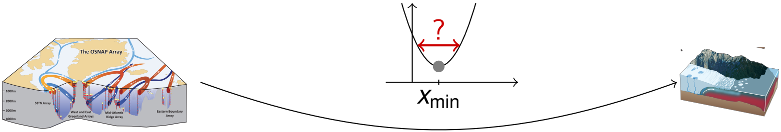

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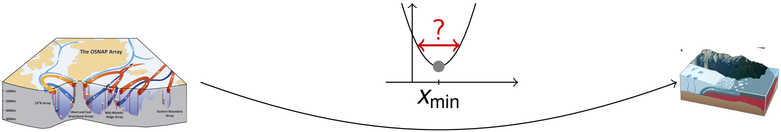


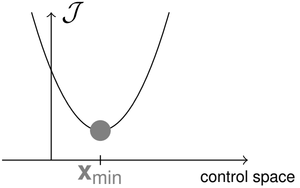
Objectives:

1. To highlight the link between UQ and physical oceanography
2. To explore UQ in oceanographic inverse problems for simplified observing systems

Uncertainty Quantification in Inverse Problems

Nonlinear inverse/optimization problem (4D-Var): Minimize

$$\mathcal{J}(\mathbf{x}) = \frac{1}{2}\Big[\underbrace{(\mathbf{y}^o - \mathcal{H}(\mathbf{x}))^T\, \mathbf{R}^{-1}\, (\mathbf{y}^o - \mathcal{H}(\mathbf{x}))}_{\text{weighted data-model misfit}} + \underbrace{(\mathbf{x} - \mathbf{x}^b)^T\, \mathbf{B}^{-1}\, (\mathbf{x} - \mathbf{x}^b)}_{\text{deviation from prior guess}}\Big]$$



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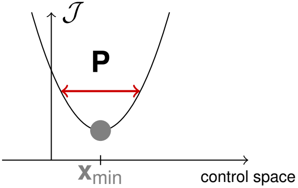

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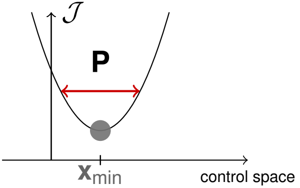

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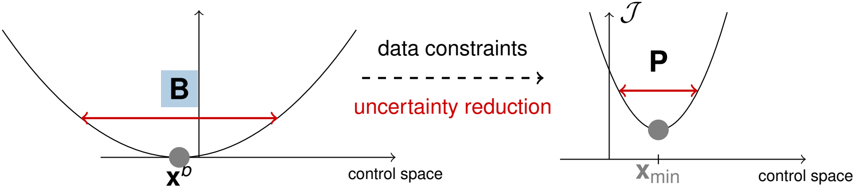

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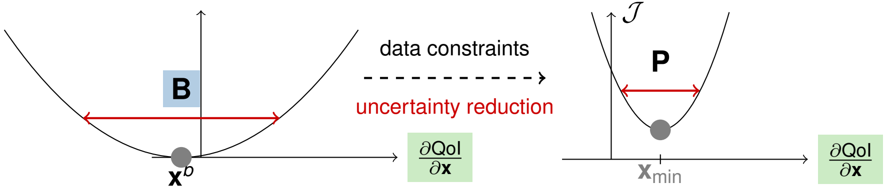

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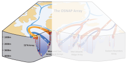
Uncertainty reduction for QoI

$$\sum_{i=1}^{M} \frac{\lambda_i}{\lambda_i + 1} \left\langle \mathbf{B}^{1/2} \frac{\partial \text{QoI}}{\partial \mathbf{x}}, \mathbf{v}_i \right\rangle^2$$

↑ data-constrained modes

$\{\mathbf{v}_i\}$ constrained by OSNAP heat transport measurements

West



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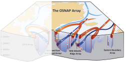
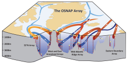


West & East



$\{\mathbf{v}_i\}$ constrained by **long-term mean** OSNAP heat transport measurements

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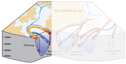

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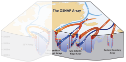

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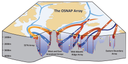

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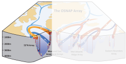

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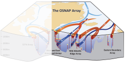

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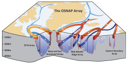

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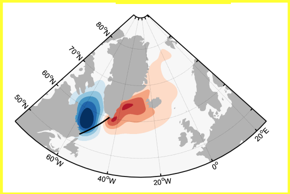

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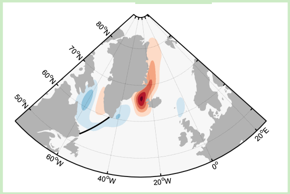

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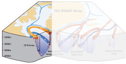

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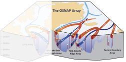

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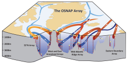

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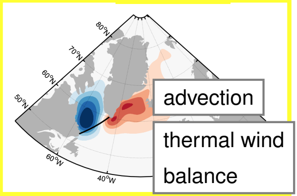

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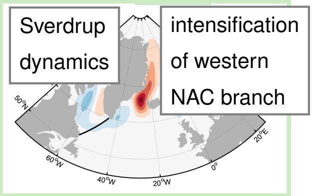

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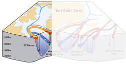
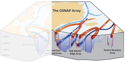
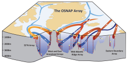


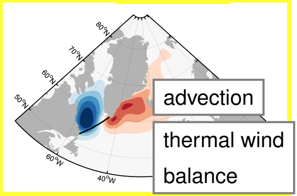
East



West & East



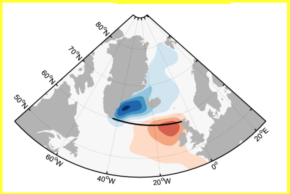
$$\mathbf{B}^{1/2}\frac{\partial \mathrm{MHT}_{\text{OSNAP-W}}}{\partial\ \mathrm{swdown}(x,y)}$$



advection

thermal wind balance

$$\mathbf{B}^{1/2}\frac{\partial \mathrm{MHT}_{\text{OSNAP-E}}}{\partial\ \mathrm{swdown}(x,y)}$$



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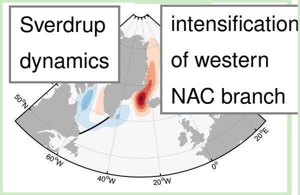
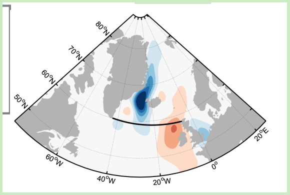


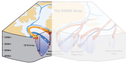
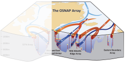
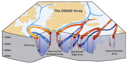
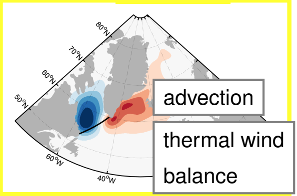
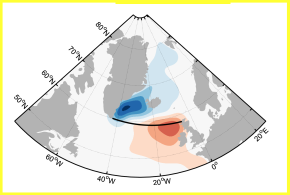
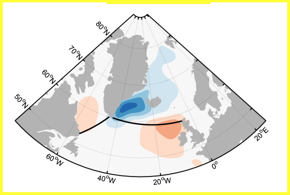
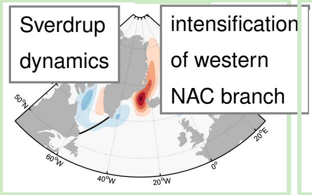
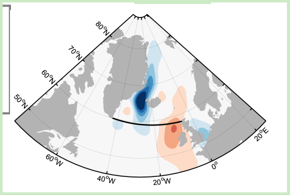
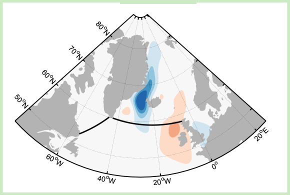
Sverdrup dynamics

intensification of western NAC branch

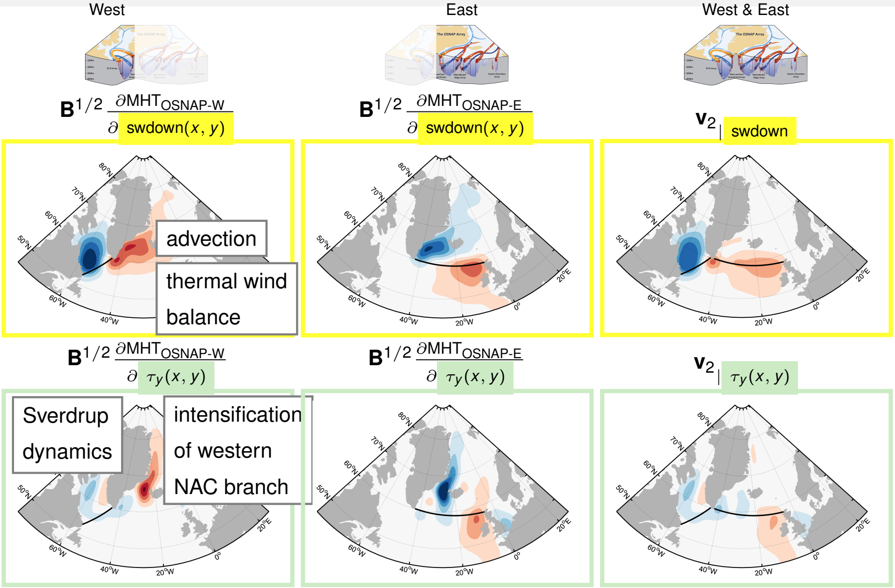
$$\mathbf{B}^{1/2}\frac{\partial \mathrm{MHT}_{\text{OSNAP-E}}}{\partial\ \tau_y(x,y)}$$



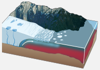
$\{\mathbf{v}_i\}$ constrained by **long-term mean** OSNAP heat transport measurements

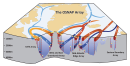
West	East	West & East
		
$\mathbf{B}^{1/2}\dfrac{\partial \mathrm{MHT}_{\text{OSNAP-W}}}{\partial\ \text{swdown}(x,y)}$	$\mathbf{B}^{1/2}\dfrac{\partial \mathrm{MHT}_{\text{OSNAP-E}}}{\partial\ \text{swdown}(x,y)}$	$\mathbf{v}_1\big\rvert_{\text{swdown(x.y)}}$
		
$\mathbf{B}^{1/2}\dfrac{\partial \mathrm{MHT}_{\text{OSNAP-W}}}{\partial\ \tau_y(x,y)}$	$\mathbf{B}^{1/2}\dfrac{\partial \mathrm{MHT}_{\text{OSNAP-E}}}{\partial\ \tau_y(x,y)}$	$\mathbf{v}_1\big\rvert_{\tau_y(x,y)}$
		

$\{\mathbf{v}_i\}$ constrained by **long-term mean** OSNAP heat transport measurements

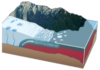


Projection $\langle \mathbf{B}^{1/2} \frac{\partial \mathrm{QoI}}{\partial \mathbf{x}}, \mathbf{v}_1 \rangle$

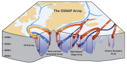


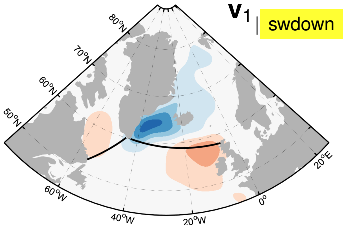


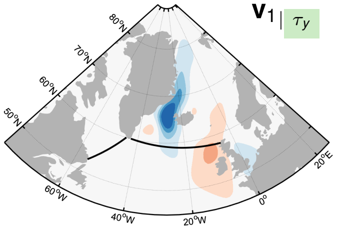
$\mathbf{B}^{1/2} \frac{\partial \mathrm{subT_{HG}}}{\partial \mathbf{x}}$



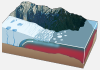
$\mathbf{v}_1$



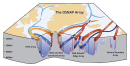




Projection $\langle \mathbf{B}^{1/2} \frac{\partial \text{QoI}}{\partial \mathbf{x}}, \mathbf{v}_1 \rangle$



$\longleftrightarrow$



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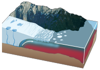
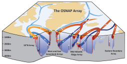
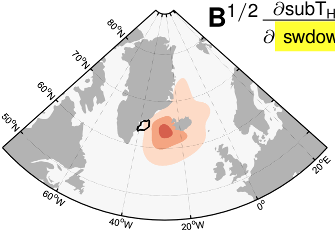


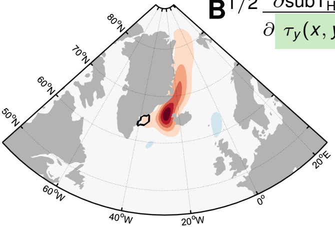
$\mathbf{v}_1$



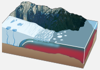
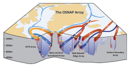
$\mathbf{B}^{1/2} \frac{\partial \text{subT}_{\text{HG}}}{\partial \text{ swdown}}$

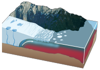


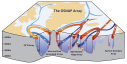
$\mathbf{B}^{1/2} \frac{\partial \text{subT}_{\text{HG}}}{\partial \ \tau_y(x,y)}$

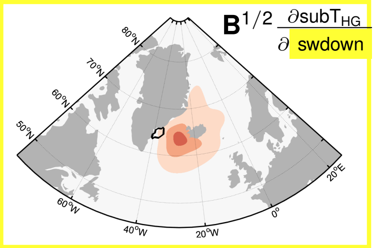


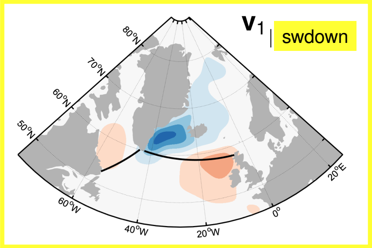
Projection $\langle \mathbf{B}^{1/2} \frac{\partial \text{QoI}}{\partial \mathbf{x}}, \mathbf{v}_1 \rangle$

 ⟷ 

$\mathbf{B}^{1/2} \frac{\partial \text{subT}_{HG}}{\partial \mathbf{x}}$ 

$\mathbf{v}_1$ 





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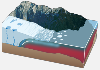

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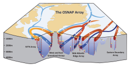

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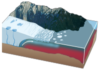
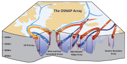


$\mathbf{v}_1$



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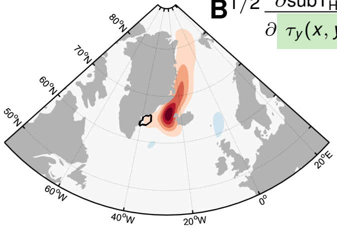
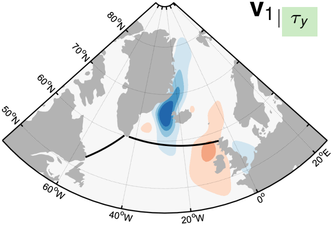
$\mathbf{B}^{1/2} \frac{\partial \mathrm{subT_{HG}}}{\partial \tau_y(x,y)}$



$\mathbf{v}_1|_{\tau_y}$

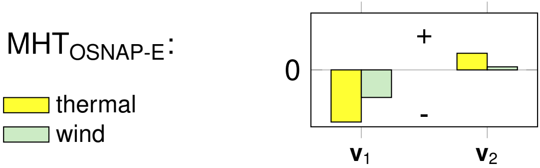


−

Constraints on Helheim subsurface temperature

by $MHT_{OSNAP\text{-}W}$ & $MHT_{OSNAP\text{-}E}$:

$$\left\langle \mathbf{B}^{1/2} \frac{\partial \mathrm{subT}_{HG}}{\partial \mathbf{x}}, \mathbf{v}_i \right\rangle \Big|$$



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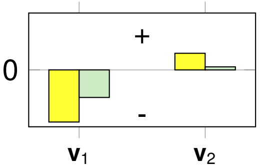
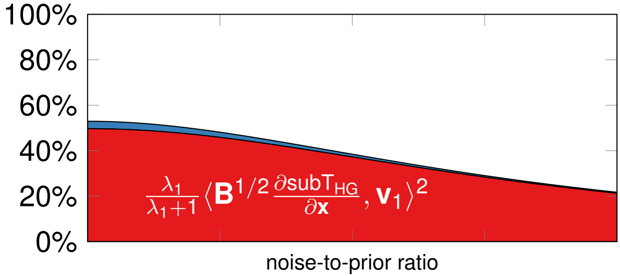
$\left\langle \mathbf{B}^{1/2} \frac{\partial \text{subT}_{\text{HG}}}{\partial \mathbf{x}}, \mathbf{v}_i \right\rangle \Big|$ thermal, wind



$$\text{uncertainty reduction} = \sum_{i=1}^{2} \frac{\lambda_i}{\lambda_i + 1} \left\langle \mathbf{B}^{1/2} \frac{\partial \text{subT}_{\text{HG}}}{\partial \mathbf{x}}, \mathbf{v}_i \right\rangle^2$$



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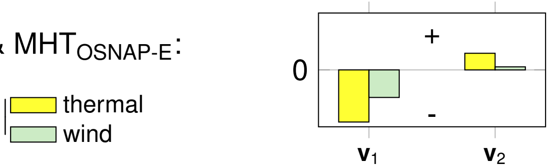

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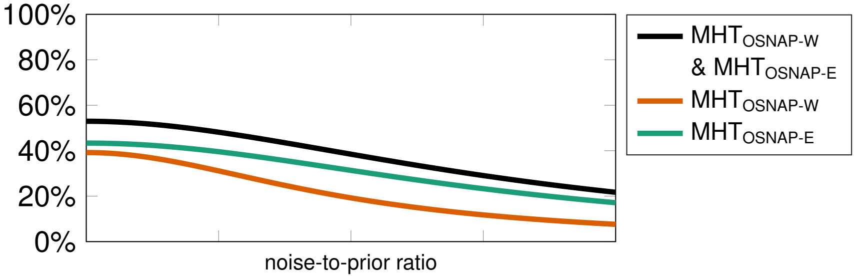
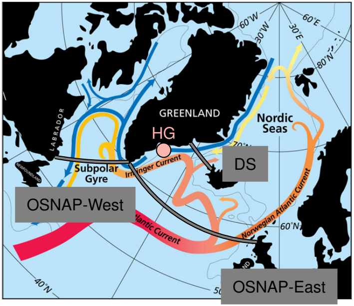
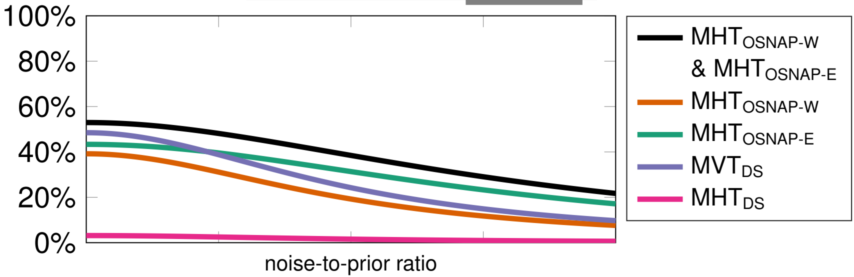


Constraints on Helheim subsurface temperature





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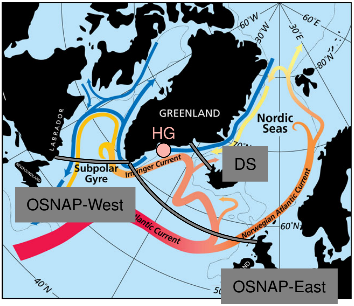

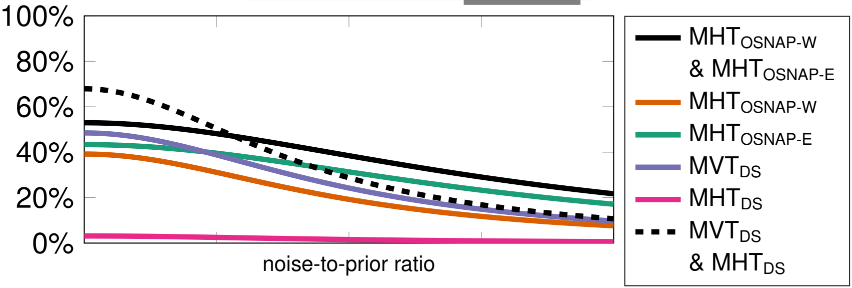

Conclusions and Outlook

- Ocean dynamics are at the core of UQ (via adjoint sensitivities)
- UQ analyzes redundancy and complementarity of data constraints

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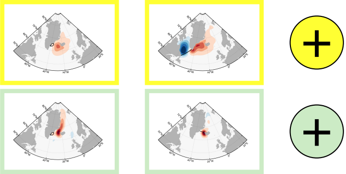

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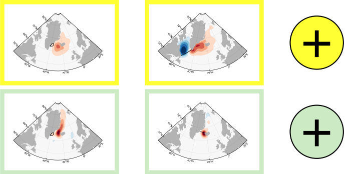

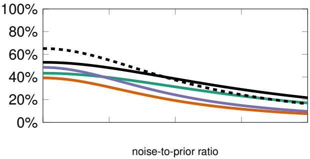

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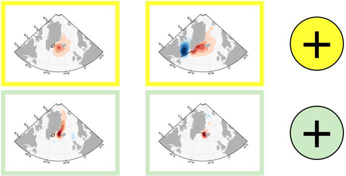
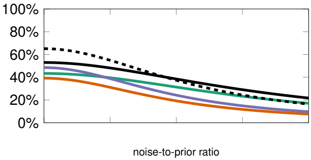
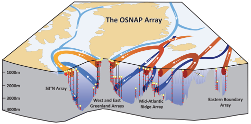




Outlook for redesign of OSNAP: Compute low-rank approximation via parallel randomized SVD methods [Halko et al., 2011]



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Halko, N., Martinsson, P. G., and Tropp, J. A. (2011).
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Nature, 504(7478):36–43.

Backup Slides

Low-rank approximation of misfit Hessian

$$\mathbf{P} = (\underbrace{\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H} + \mathbf{B}^{-1}}_{\text{linearized (or Gauss-Newton) Hessian}})^{-1}$$

Misfit Hessian $\mathbf{H}_{\text{misfit}} = \mathbf{H}^T \mathbf{R}^{-1} \mathbf{H}$ is of low rank:

$$\tilde{\mathbf{H}}_{\text{misfit}} = \mathbf{B}^{1/2}\, \mathbf{H}^T \mathbf{R}^{-1} \mathbf{H}\, \mathbf{B}^{1/2} \approx \sum_{i=1}^{M} \lambda_i \mathbf{v}_i \mathbf{v}_i^T$$

Expression for posterior covariance:

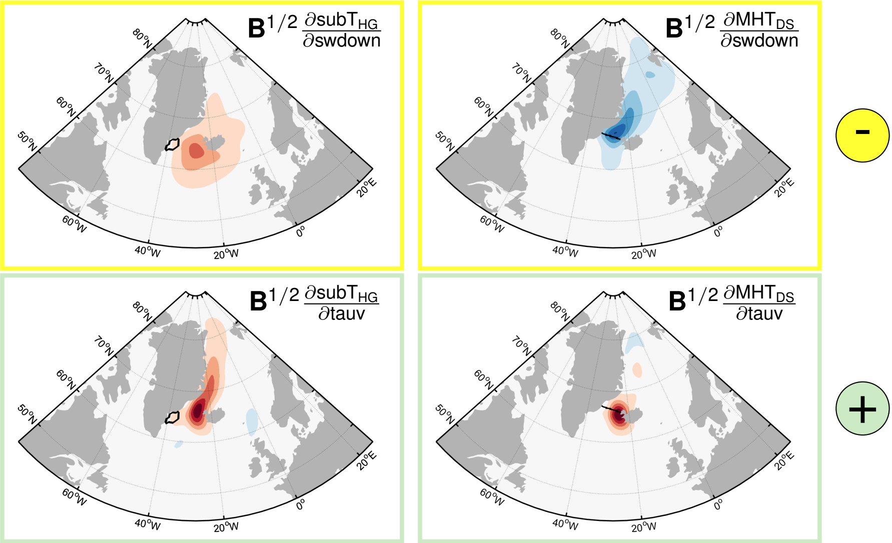
$$\mathbf{P} = \mathbf{B}^{1/2} (\tilde{\mathbf{H}}_{\text{misfit}} + \mathbf{I})^{-1} \mathbf{B}^{1/2} = \mathbf{B}^{1/2} (\mathbf{I} - \sum_{i=1}^{M} \frac{\lambda_i}{\lambda_i + 1} \mathbf{v}_i \mathbf{v}_i^T)\, \mathbf{B}^{1/2}$$

$$= \mathbf{B} - \sum_{i=1}^{M} \frac{\lambda_i}{\lambda_i + 1} \left(\mathbf{B}^{1/2}\mathbf{v}_i\right) \left(\mathbf{B}^{1/2}\mathbf{v}_i\right)^T.$$

Projection of weighted adjoint sensitivities - DS

$\mathbf{B}^{1/2}\frac{\partial \mathrm{subT_{HG}}}{\partial \mathbf{x}}$

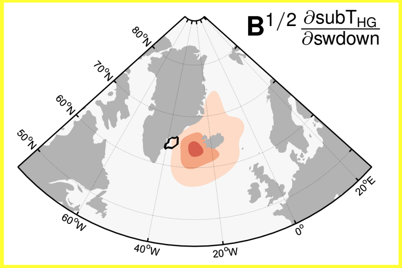
$\mathbf{B}^{1/2}\frac{\partial \mathrm{MHT_{DS}}}{\partial \mathbf{x}}$

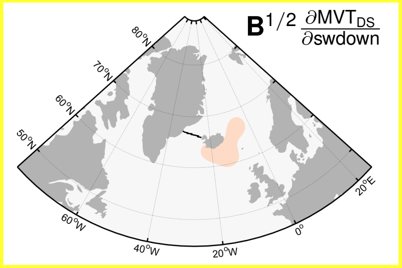


Projection of weighted adjoint sensitivities - DS

$\mathbf{B}^{1/2}\frac{\partial \mathrm{subT_{HG}}}{\partial \mathbf{x}}$

$\mathbf{B}^{1/2}\frac{\partial \mathrm{MVT_{DS}}}{\partial \mathbf{x}}$





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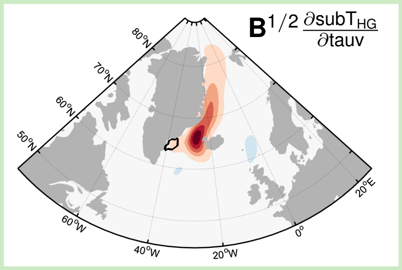

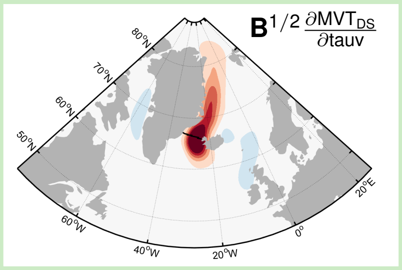

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