EFSO and DFS diagnostics for JMA’s global Data Assimilation System: their caveats and potential pitfalls

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Special thanks to:

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1. Adjoint and Ensemble FSO

FSO: Forecast Sensitivity to Observations

- $J(\mathbf{e}) = \mathbf{e}^T\mathbf{C}\mathbf{e}$, **e**: vector of forecast error
- $\Delta J = \mathbf{e}_{t|0}^T\mathbf{C}\mathbf{e}_{t|0} - \mathbf{e}_{t|-6}^T\mathbf{C}\mathbf{e}_{t|-6}$

$$\approx \mathbf{d}^{\mathrm{O-B}^T}\left(\frac{\partial \mathbf{x}_0^a}{\partial \mathbf{y}^{\mathrm{o}}}\right)\left(\frac{\partial \mathbf{x}_{t|0}^f}{\partial \mathbf{x}_0^a}\right)\left(\frac{\partial J}{\partial \mathbf{x}_{t|0}^f}\right)\Bigg|_{(\mathbf{x}_{t|0}^f+\mathbf{x}_{t|-6}^f)/2}$$

$$= \mathbf{d}^{\mathrm{O-B}^T} \quad \mathbf{K}^T \quad \mathbf{M}^T \quad \mathbf{C}\left(\mathbf{e}_{t|0} + \mathbf{e}_{t|-6}\right)$$

Adjoint FSO (Langland and Baker 2004)

$$\approx \mathbf{d}^{\mathrm{O-B}^T} \frac{1}{K-1} \mathbf{R}^{-1}\mathbf{Y}_0^a\mathbf{X}_{t|0}^{fT} \ \mathbf{C}\left(\mathbf{e}_{t|0} + \mathbf{e}_{t|-6}\right)$$

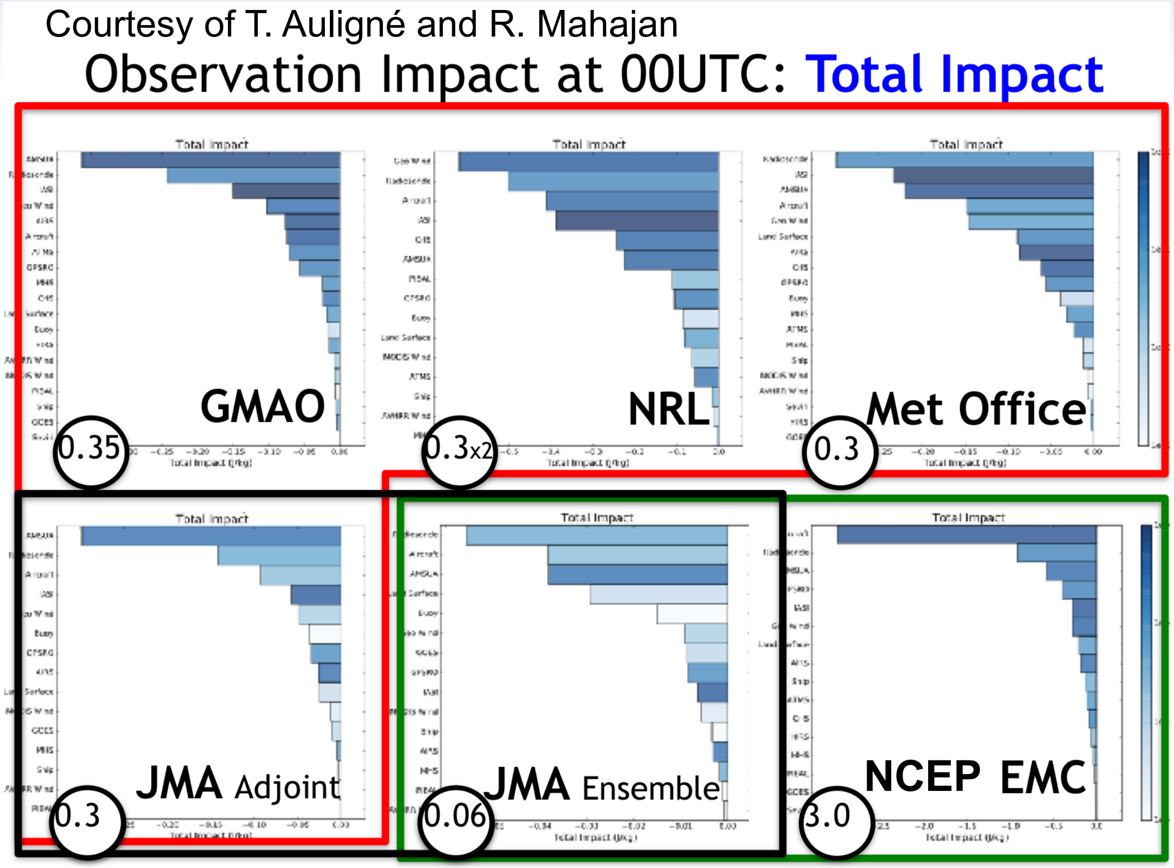
Ensemble FSO (Kalnay et al. 2012)

$$\because \mathbf{MK} = \frac{1}{K-1}\mathbf{M}\mathbf{X}_0^a\mathbf{X}_0^{aT}\mathbf{H}^T\mathbf{R}^{-1} \approx \frac{1}{K-1}\mathbf{X}_{t|0}^f\mathbf{Y}_0^{aT}\mathbf{R}^{-1}$$

2. FSOI inter-comparison project
(c.f. Rahul Mahajan's talk this morning)

- Different global NWP centers computed FSOI data for the same period using the same error-norm metric.
- Data collected from
  - GMAO, NRL, Met Office, JMA (adjoint)
  - NCEP, JMA (ensemble)
- Note: JMA is the only center that provided both adjoint and ensemble FSO.

3. EFSO impact amplitude deviates from adjoint FSO



- Adjoint FSO from different centers have comparative amplitudes, whereas
- NCEP (EMC)'s EFSO exhibits O(10) larger amplitude, and
- JMA's EFSO exhibits ~ 0.2 times smaller amplitude
- Why?

4. NCEP's EFSO Overestimation problem:

Inconsistent use of **K** for mean update and covariance update

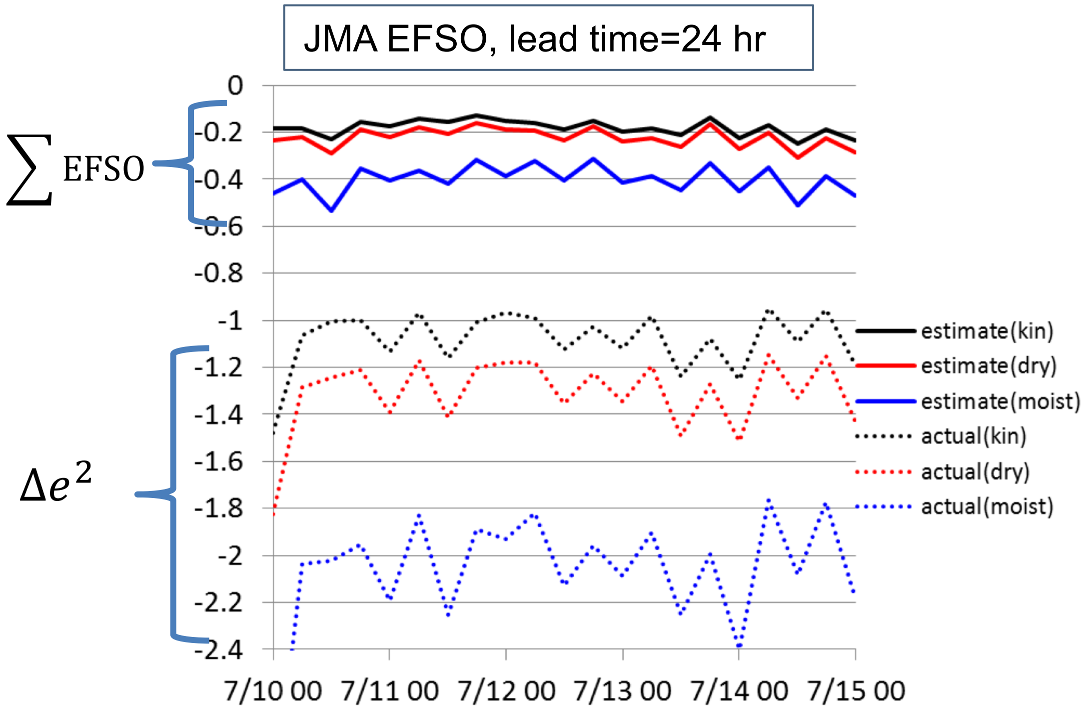
- Mean update:
  - Compute $\mathbf{P}^a$ first, then compute $\delta\mathbf{x}^a$ by $\mathbf{Kd}=\mathbf{P}^a\mathbf{H}^T\mathbf{R}^{-1}\mathbf{d}$
  - No inflation applied to $\mathbf{P}^a$
- Covariance (perturbation) update:
  - Compute $\mathbf{P}^a$, then applied posterior inflation (relaxation to prior)

→ To correctly estimate obs impact (how each obs improved ens mean), $\mathbf{X}^f$ has to be initialized from un-inflated $\mathbf{X}^a$.

- But the NCEP implementation of EFSO uses $\mathbf{X}^f$ initialized from inflated $\mathbf{X}^a$ (D. Groff., pers. comm.)

5. JMA's EFSO Underestimation problem: Estimated and actual forecast error reduction

$$\sum \mathrm{EFSO} \qquad \Delta e^2 = \frac{1}{2}\mathbf{e}_{t|0}^{f\mathrm{T}}\mathbf{C}\mathbf{e}_{t|0}^{f} - \frac{1}{2}\mathbf{e}_{t|-6}^{f\mathrm{T}}\mathbf{C}\mathbf{e}_{t|-6}^{f}$$



- EFSO successfully reproduces temporal variation of forecast error reductions (correlation coefficient as high as ~ 0.8), but
- Only ~ 20 % of the amplitude explained by EFSO.

6. A possible reason for impact underestimation (1/3)

- EFSO implemented for JMA's LETKF underestimates forecast error reduction.
- Why?
- Bug? → not found.
- Possible reason: **forecast errors not well captured by the space spanned by the forecast perturbations**

6. A possible reason for impact underestimation (2/3)

- EFSO formulation: $\Delta e^{f-g} \approx \frac{1}{K-1}\mathbf{d}^T\mathbf{R}^{-1}\left[\rho \circ \mathbf{Y}^a\,\mathbf{X}^{f^T}\right]\mathbf{C}(\mathbf{e}^f_{t|0}+\mathbf{e}^f_{t|-6})$

- In evaluating

$$\mathbf{X}^{f^T}\mathbf{C}(\mathbf{e}^f_{t|0}+\mathbf{e}^f_{t|-6}) = \underbrace{\left(\mathbf{C}^{1/2}\mathbf{X}^f\right)}_{\widetilde{\mathbf{X}}^f}{}^T \underbrace{[\mathbf{C}^{\frac{1}{2}}(\mathbf{e}^f_{t|0}+\mathbf{e}^f_{t|-6})]}_{\tilde{\mathbf{e}}} =: \widetilde{\mathbf{X}}^{f^T}\tilde{\mathbf{e}}$$

  the portion of $\tilde{\mathbf{e}}$ that lies in the nullspace of $\widetilde{\mathbf{X}}^f$ does not contribute to $\widetilde{\mathbf{X}}^{f^T}\tilde{\mathbf{e}}$.

  Namely:

- Let $\tilde{\mathbf{e}} = \tilde{\mathbf{e}}_{\mathrm{span}} + \tilde{\mathbf{e}}_{\mathrm{null}}, \tilde{\mathbf{e}}_{\mathrm{span}} \in \mathrm{span}\left(\widetilde{\mathbf{X}}^f\right), \tilde{\mathbf{e}}_{\mathrm{null}} \in \mathrm{null}\left(\widetilde{\mathbf{X}}^f\right)$

  then

$$\widetilde{\mathbf{X}}^{f^T}\tilde{\mathbf{e}} = \widetilde{\mathbf{X}}^{f^T}\left(\tilde{\mathbf{e}}_{\mathrm{span}} + \tilde{\mathbf{e}}_{\mathrm{null}}\right) = \widetilde{\mathbf{X}}^{f^T}\tilde{\mathbf{e}}_{\mathrm{span}}$$

- N.B.: This issue does not arise in adjoint FSO.

6. A possible reason for impact underestimation (3/3)

- Does this hypothesis really explain the impact underestimation?

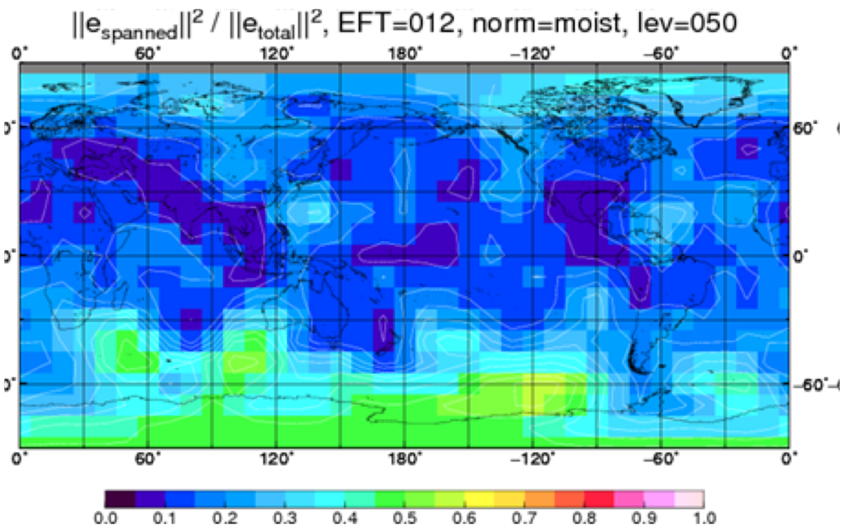
→ Verify the hypothesis by performing the following diagnostics:

- For each model grid,
  - Restrict all state vectors (mean and ptb) into localization volume
  - Decompose $\tilde{\mathbf{e}}$ into $\tilde{\mathbf{e}}_{\mathrm{span}}$ and $\tilde{\mathbf{e}}_{\mathrm{null}}$. (detail in the backup slide)
  - Compute the "explained fraction" $\frac{\|\tilde{\mathbf{e}}_{\mathrm{span}}\|^2}{\|\tilde{\mathbf{e}}\|^2}$.
  - Compare this with the impact underestimation $\frac{\sum \mathrm{EFSO}}{\Delta e^2}$.
- If the two agrees, we conclude that the hypothesis is likely correct.

Diagnosed “explained fraction” $\frac{\|\tilde{\mathbf{e}}_{\text{span}}\|^2}{\|\tilde{\mathbf{e}}\|^2}$

Horizontal distribution

(near tropopause level)



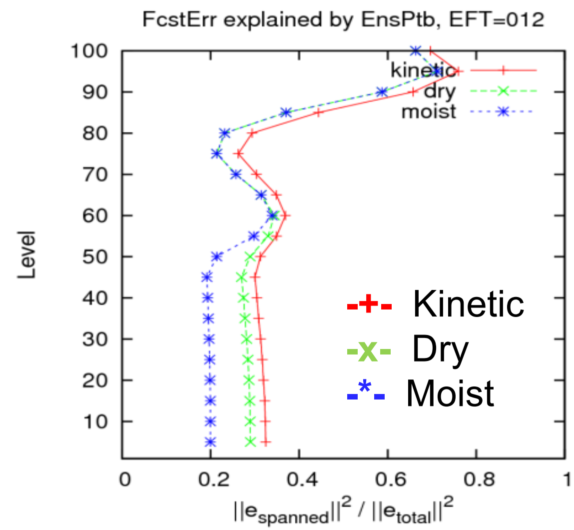
- Fcst err well-captured by ensemble over the SH ocean, but not over the land.

→ Perhaps related to observation density:

  – Data-sparse area: analysis (verification) and forecast both close to model’s free-run → $\mathbf{e}^f_{t|0}$ similar to Bred Vector → covered well by $\mathbf{X}^f$

Vertical Profile

(global average)



- Errors in moisture difficult to capture by the ensemble.

Very good agreement between $\frac{\|\tilde{\mathbf{e}}_{\text{span}}\|^2}{\|\tilde{\mathbf{e}}\|^2}$ and $\frac{\sum \text{EFSO}}{\Delta e^2}$! (both ~ 20%)

" Explained fraction" increases when ens covariance is given higher weight

- The 20 % "explained fraction" is obtained for EFSO within hybrid 4D-Var (LETKF anl mean recentered to Var anl) where

$$\mathbf{B}^{hyb}=0.23\mathbf{B}^{ens}+0.77\mathbf{B}^{clim}$$

- We observed that "explained fraction" increases monotonically with ens cov weight.
  - "Explained fraction" for pure (stand-alone) LETKF was as high as 67%

7. Conclusion

- EFSO is successfully implemented on JMA’s global DA system,
- but the total impact considerably underestimated.
- “Explaiend fraction” diagnostics has been proposed that decomposes fcst err into column- and null- spaces of the fcst ensemble $\mathbf{X}^f$
- The results suggests that significant portion of fcst err lies in the null-space of $\mathbf{X}^f$,
- which exposes the lack of the ensemble size used at JMA (currently only 50).

Questions? Suggestions?

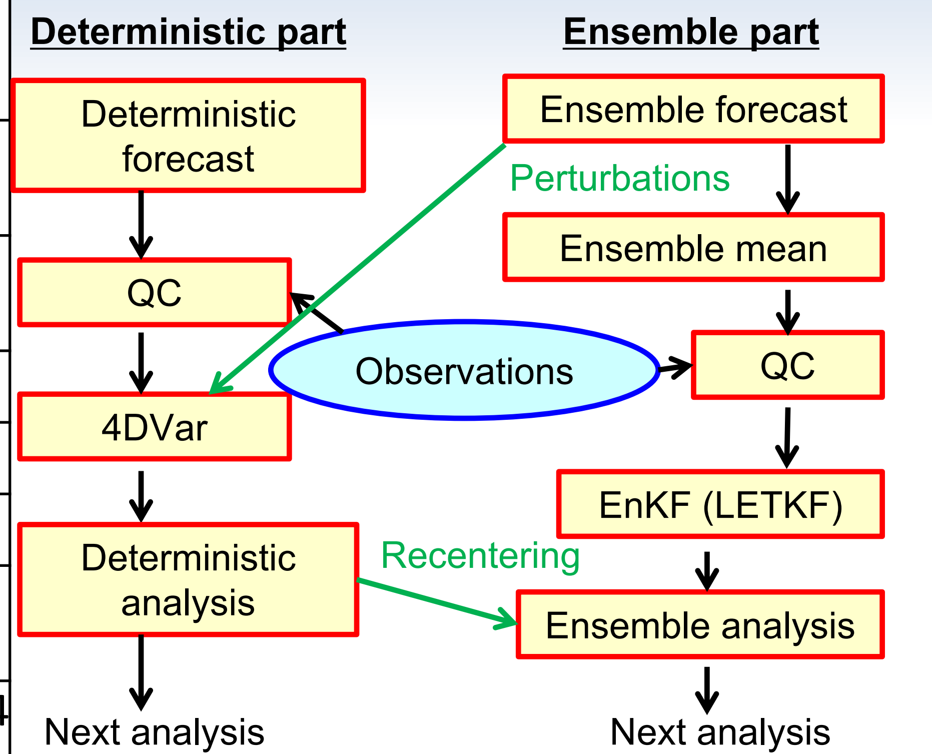
Questions from me to you:

- Have I addressed localization correctly?
- What does my conclusion imply about validity of EFSO diagnostics?

Backup slides

Hybrid 4DVar-LETKF DA developed in JMA

Analysis resolution (outer / inner)	T_L959L100 (~20km, top:0.01hPa) / T_L319L100 (~55km, top:0.01hPa)
Assimilation window	6 hours (analysis time +/- 3 hours)
Hybrid method	Extended control variable method (Lorenc 2003)
Weights on $\mathbf{B}$	$\beta_{stat}{}^2 = 0.77$, $\beta_{ens}{}^2 = 0.23$
LETKF resolution	T_L319L100
Ensemble size	50
Localization scale (4DVar)	Horizontal: 800km Vertical: 0.8 scale heights
Localization scale (LETKF)	Horizontal: 400km, Vertical: 0.4 (0.8 for Ps) scale heights
Covariance inflation	Adaptive inflation (Miyoshi 2011)



$$\mathbf{B} = \beta_{stat}^2 \mathbf{B}_{stat} + \beta_{ens}^2 \mathbf{B}_{ens}$$

Static (Climatological) background error covariance

Ensemble-based background error covariance

Operational global DA at JMA is 4DVar (not hybrid)

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EFSO implementation at JMA

by Yoichiro Ota (2015)

- DA system: hybrid LETKF/4D-Var coupled with JMA GSM
  - Resolution: (outer) TL959L100 ; (inner and ensemble) T319L100
  - Window: 6 hours (analysis time +/- 3 hours)
  - **B** weights: 77% from static, 23% from ensemble
  - Member size: 50
  - Localization scales (e-folding):
    - LETKF: Horizontal: 400km, Vertical: 0.4 scale heights
    - 4D-Var: Horizontal: 800km, Vertical: 0.8 scale heights
  - Covariance Inflation: Adaptive inflation of Miyoshi (2011)
  - LETKF part initially coded by Dr. T. Miyoshi; maintained and updated by Y. Ota and T. Kadowaki.
- EFSO:
  - Lead-times investigated: FT=0,6,12,24
  - Localization scales: same as LEKTF
    - advection: “moving localization scheme” of Ota et al.(2013) with scaling factor of 0.5 for horizontal wind.
  - Verification: high-resolution analysis from 4D-Var
  - Error norm: KE, Dry TE and Moist TE
- Period: Jul. 10, 2013, 06UTC – Jul. 15, 2013, 18UTC (5days, 20cases)

Decomposition of fcst error into column- and null- spaces of fcst ptbs

- Fix a grid and consider a local patch that would be used if an observation was located at the grid point in question. In the derivation below, all vectors/matrices are assumed to be restricted to this local patch.
- In EnKF, the sum of each column of $\mathbf{X}^f$ is zero, so rank($\mathbf{X}^f$)=$K-1$: $\mathrm{span}(\widetilde{\mathbf{X}}^f) =$ $\mathrm{span}\left([\widetilde{\mathbf{X}}^f]_1, \cdots, [\widetilde{\mathbf{X}}^f]_{K-1}, [\widetilde{\mathbf{X}}^f]_K\right) = \mathrm{span}\left([\widetilde{\mathbf{X}}^f]_1, \cdots, [\widetilde{\mathbf{X}}^f]_{K-1}\right)$

In light of this, we now denote by $\widetilde{\mathbf{X}}^f$ the first $K-1$ columns of the original $\widetilde{\mathbf{X}}^f$.

- Now, suppose that $\widetilde{\mathbf{e}} := \mathbf{C}^{\frac{1}{2}}(\mathbf{e}^f_{t|0}+\mathbf{e}^f_{t|-6})$ can be decomposed as
- $\widetilde{\mathbf{e}} = \widetilde{\mathbf{e}}_{\mathrm{span}} + \widetilde{\mathbf{e}}_{\mathrm{null}}$, $\widetilde{\mathbf{e}}_{\mathrm{span}} = \sum_{\boldsymbol{k}=\mathbf{1}}^{\boldsymbol{K}-\mathbf{1}} \alpha_k [\widetilde{\mathbf{X}}^f]_{\boldsymbol{k}} = \widetilde{\mathbf{X}}^f \boldsymbol{\alpha}$,

$\boldsymbol{\alpha} = (\alpha_1, \quad \cdots \quad \alpha_{K-1})^T$

Multiplying $\widetilde{\mathbf{e}} = \widetilde{\mathbf{e}}_{\mathrm{span}} + \widetilde{\mathbf{e}}_{\mathrm{null}}$ with $\left(\mathbf{C}^{1/2}\mathbf{X}^f\right)^T =: \widetilde{\mathbf{X}}^{f^T}$ from left, $\widetilde{\mathbf{e}}_{\mathrm{null}}$ vanishes by definition , giving:

$$\widetilde{\mathbf{X}}^{f^T} \widetilde{\mathbf{e}} = \widetilde{\mathbf{X}}^{f^T} \left(\widetilde{\mathbf{X}}^f \boldsymbol{\alpha} + \widetilde{\mathbf{e}}_{\mathrm{null}}\right) = \widetilde{\mathbf{X}}^{f^T} \widetilde{\mathbf{X}}^f \boldsymbol{\alpha}$$

$$\therefore \boldsymbol{\alpha} = \left(\widetilde{\mathbf{X}}^{f^T} \widetilde{\mathbf{X}}^f\right)^{-1} \widetilde{\mathbf{X}}^{f^T} \widetilde{\mathbf{e}}$$

- Once $\boldsymbol{\alpha}$ is determined, we can obtain $\|\widetilde{\mathbf{e}}_{\mathrm{span}}\|^2$ and $\|\widetilde{\mathbf{e}}_{\mathrm{null}}\|^2$ by

$$\|\widetilde{\mathbf{e}}_{\mathrm{span}}\|^2 = \|\widetilde{\mathbf{X}}^f \boldsymbol{\alpha}\|^2 = (\widetilde{\mathbf{X}}^f \boldsymbol{\alpha})^T (\widetilde{\mathbf{X}}^f \boldsymbol{\alpha}) = \boldsymbol{\alpha}^T \widetilde{\mathbf{X}}^{f^T} \widetilde{\mathbf{X}}^f \boldsymbol{\alpha} = \boldsymbol{\alpha}^T \widetilde{\mathbf{X}}^{f^T} \widetilde{\mathbf{e}}$$

$$\|\widetilde{\mathbf{e}}_{\mathrm{null}}\|^2 = \|\widetilde{\mathbf{e}}\|^2 - \|\widetilde{\mathbf{e}}_{\mathrm{span}}\|^2$$