# Accounting for Error due to Unresolved Scales in Data Assimilation

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## 1. Introduction

In atmospheric data assimilation, observations of the atmospheric state will include scales of motion unable to be resolved by numerical models assimilating these observations. The resulting error caused by this scale-mismatch is called the error due to unresolved scales and is a component of the representation error. The standard approach to dealing with this error is to include it as part of the observation weighting matrix. However, other approaches exist which take explicit account of resolved and unresolved scales such as the Schmidt-Kalman Filter (Janjić and Cohn, 2006).

## 2. Aims

**The aim of this poster is to demonstrate the Schmidt-Kalman Filter's (SKF) ability to compensate for error due to unresolved scales**. It does this through:

- Use of correct initial forecast error covariance $\boldsymbol{P}_0^f$, observation error covariance $\boldsymbol{R}$, model error covariance $\boldsymbol{Q}$, forecast model $\boldsymbol{F}$ and observation operator $\boldsymbol{H}$.
- Disregarding the unresolved state values to reduce computations.
- Accounting for state and time dependence of the representation error.

The filters we will compare the SKF to are:

1. The Optimal Kalman Filter (OKF). This filter uses all correct parameters and evolves the resolved and unresolved states.
2. The Reduced-Order Kalman Filter (RKF). This filter uses only the red parameter elements displayed below. It is essentially a Kalman Filter for the resolved state only and represents how Kalman Filters are currently applied.

## 3. A Simple Example

Our model is a simple two variable dynamical system with one variable each for the resolved and unresolved scales while. Both processes will be Gaussian random walks with the unresolved state having a contribution from the resolved scales. The initial conditions for the states are chosen so the resolved state is an order of magnitude larger than the unresolved state.

The model parameters are (Brown et al., 2012)

$$\boldsymbol{F} = \begin{bmatrix} 1 & 0 \\ 0.05 & e^{-1/2} \end{bmatrix}, \quad \boldsymbol{P}_0^f = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{Q} = \begin{bmatrix} 1 & 0 \\ 0 & e^{-1} \end{bmatrix},$$

$$\boldsymbol{H} = [1 \quad 1], \qquad \boldsymbol{R} = 1.$$


### References

1. Janjić, T. and Cohn, S.E., 2006. Treatment of observation error due to unresolved scales in atmospheric data assimilation. *Monthly Weather Review*, *134*(10),
2. Brown, R.G. and Hwang, P.Y., 2012. *Introduction to random signals and applied Kalman filtering* (Vol. 4). New York: Wiley. Page 192



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## 4. Numerical Results

### Resolved State Evolution

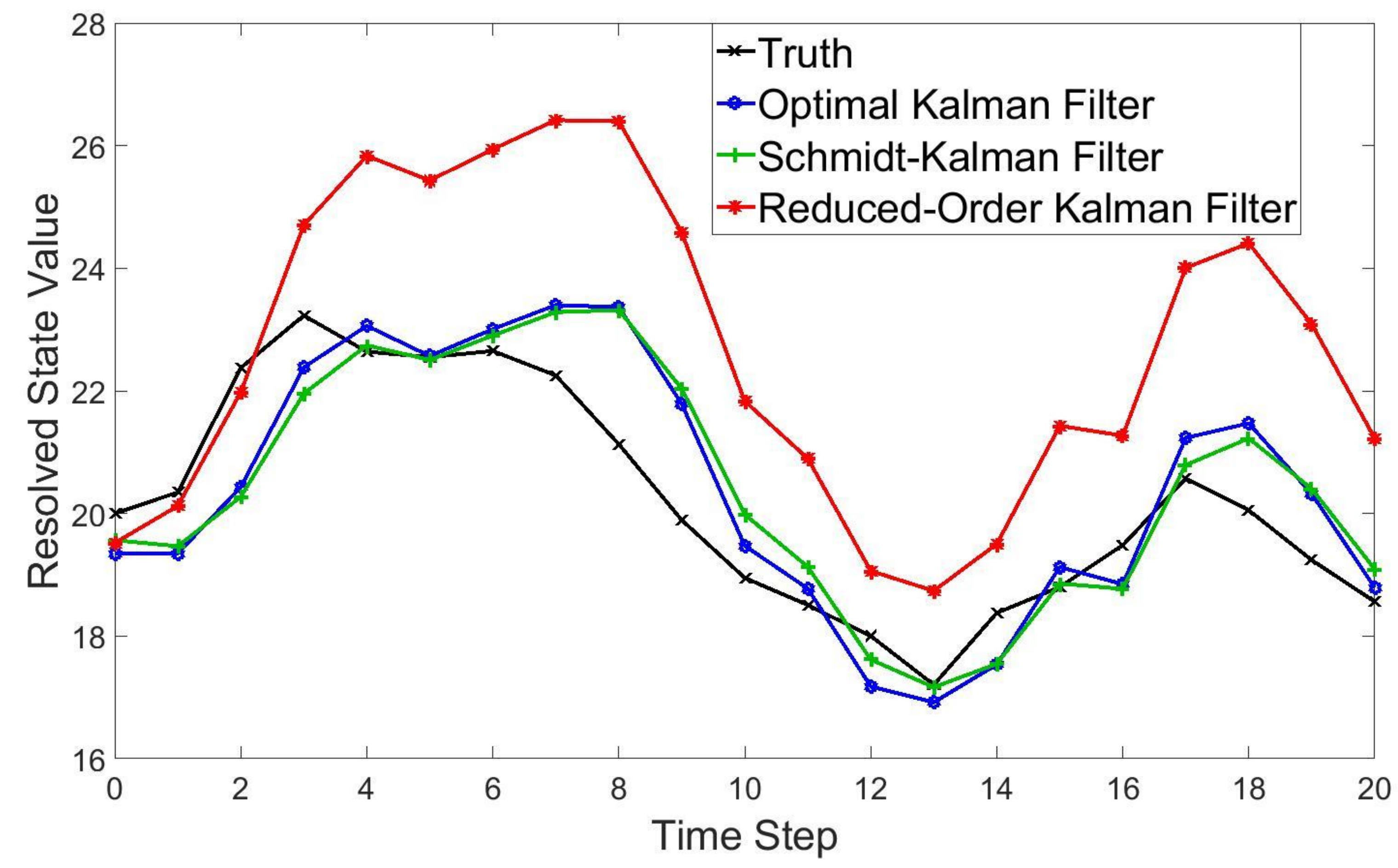


**Figure 1**: The evolution of the resolved state analysis on a typical run for the three filters compared to the truth trajectory.

- The mean-square-errors of the OKF, SKF and RKF are1.0528, 1.1142 and 8.5977 respectively.
- The OKF has only slightly less error than the SKF despite the extra computational expense.
- The RKF has a large error as it doesn't account for the unresolved scales.

### Resolved State Variance

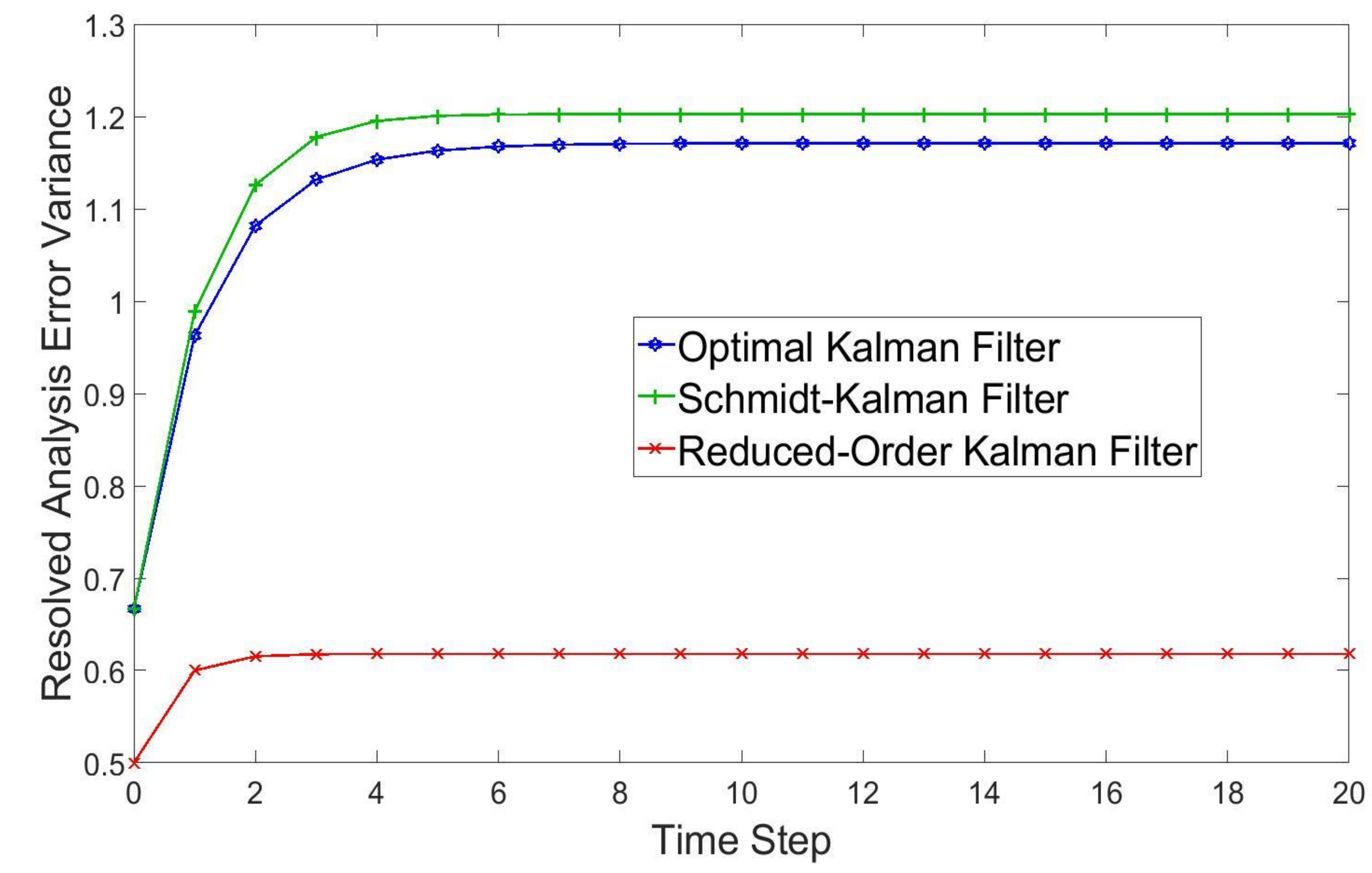


**Figure 2**: The evolution of the resolved state variance for the three filters.

- The variance of the OKF and SKF are in agreement with their mean-square-errors.
- The RKF's variance is in disagreement with its mean-square-error as the unresolved paraeters aren't correctly accounted for.

## 5. Conclusion

The SKF is a suboptimal filter which sacrifices estimation performance for computational performance. We can also see that the SKF can account for unresolved scales error. Future plans for testing this filter include comparisons with other methods of accounting for unresolved scales error and using approximations for unknown parameters within an atmospheric context.