Final Report

Optimal Trajectory Planning Algorithm for Connected and Autonomous Vehicles Towards Uncertainty of Actuated Traffic Signals

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16. Abstract

This report introduces a robust green light optimal speed advisory (GLOSA) system for fixed and actuated traffic signals which considers a probability distribution. These distributions represent the domain of possible switching times from the signal phasing and timing (SPaT) messages. The system finds the least-cost (minimum fuel consumption) vehicle trajectory using a computationally efficient A* algorithm incorporated within a dynamic programming (DP) procedure to minimize the vehicle's total fuel consumption. Constraints are introduced to ensure that vehicles do not collide with other vehicles, run red indications, or exceed a maximum vehicular jerk for passenger comfort. Results of simulation scenarios are evaluated against empirical comparable trajectories of uninformed drivers to compute fuel consumption savings. The proposed approach produced significant fuel savings compared to an uninformed driver behavior, amounting to 37% on average for deterministic SPaT and 30% for stochastic SPaT data. A sensitivity analysis was performed to understand how the degree of uncertainty in SPaT predictions affects the optimal trajectory's fuel consumption. The results present the required levels of confidence in these predictions to achieve savings in fuel consumption. Specifically, the study demonstrates that the proposed system can be within 85% of the maximum savings if the timing error is (±3.3 seconds) at a 95% confidence level. Results also emphasize the importance of more reliable SPaT predictions as the time to green decreases relative to the time the vehicle is expected to reach the intersection given its current speed.

17. Key Words: Actuated traffic signals, Optimizing vehicle trajectories, Uncertain switching times, Eco-driving, Stochastic optimization.	**18. Distribution Statement**

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1. INTRODUCTION

The transportation sector is the main petroleum consumer in the US. The US Energy Information Administration estimates that the share of primary energy consumed by the transportation sector is 26% of total consumption (*Monthly Energy Review*, 2021). This results in an even larger share of greenhouse gas production at 35% of total emissions (EPA). These estimates are not likely to decline significantly in the next decade (Mousa et al., 2020). Furthermore, a significant portion of fuel consumed by vehicles is wasted at signalized intersections where automobiles come to a full stop, idle, and accelerate back to the desired speed. Total annual fuel losses at intersections is estimated to be about 2.8 billion gallons of gasoline per year (David Schrank et al.), which constitutes about 7% of total energy consumption in the US. To address this problem, several research efforts have developed so-called "eco-driving" systems that aim to optimize the speed profile of vehicles approaching signalized intersections. Although eco-driving systems have existed since the early 1990s, their implementation was limited to simple practices such as keeping tire pressure at optimum levels and smoothly accelerating or decelerating.

Many transportation studies have since used more sophisticated techniques to decrease the consumption of fossil fuels and greenhouse gas emissions to reduce their disastrous effects on the environment. Optimizing vehicle speed profiles on the roads is one such way to produce significant fuel savings. Moreover, vehicle speed profiles at signalized intersections warrant further study due to a large amount of fuel wasted at intersections. In this report, an overview of the vehicle trajectory optimization problem in the vicinity of actuated signals is described, and a review of the previous research work is presented. In the next sections, a detailed definition of the problem is provided, and a thorough review of the literature is presented illustrating the state of the art in this field of research.

1.1. Eco-driving at Actuated Traffic Signals

Even if the time it takes for actuated traffic lights to switch is uncertain, there are still ways to use this information to save fuel. (Mahler & Vahidi, 2014) created an algorithm to plan a vehicle's optimal speed while approaching actuated traffic signals. This algorithm uses both historical data and real-time phase information to predict when lights will change. To simplify the calculations, the algorithm uses a simplified cost function instead of a more complicated fuel consumption model. By using this algorithm, fuel savings of up to 6% were achieved over drivers who were not informed about the signals.

(Sun et al., 2020) developed a system for eco-driving for vehicles that travel through multiple actuated traffic signal controllers where the Signal Phasing and Timing (SPaT) information is uncertain. They used an effective red-light duration (ERD) concept to address randomness in signal switching times and formulated the problem as a chance-constrained stochastic program. They used Dynamic Programming (DP) to optimize the vehicle speed, and they achieved fuel savings of approximately 50-57% with minimal impact on arrival times. However, the study's major drawback is that it uses general probability density functions to account for uncertain switching times for different traffic conditions instead of using real-world SPaT information. Additionally, the fuel consumption model used in the study is computationally complex since it considers engine and transmission torque and gear number, making it impractical for real-time applications. Despite these limitations, previous literature suggests that it is possible to improve eco-driving at signalized intersections using SPaT information without the need for significant infrastructure changes.

2. PROBLEM DEFINITION

The eco-driving system uses communication systems to transmit SPaT information from the signal infrastructure to nearby vehicles using Dedicated Short-Range Communications (DSRC). This communication takes place in a perimeter of approximately 300 meters away from the transmitting roadside unit (RU). DSRC can be either vehicle-to-vehicle (V2V), vehicle-to-infrastructure (V2I), or infrastructure-to-vehicle (I2V). The eco-driving system can be implemented for both automated vehicles and human-driven vehicles. For automated vehicles, the vehicle modifies its trajectory automatically, while in human-driven vehicles, the system works as a speed advisory system for drivers (Kamalanathsharma & Rakha, 2014).

Vehicle trajectory optimization is the process of modifying a vehicle's speed profile to optimize fuel consumption and emissions. For example, when a vehicle approaches an actuated signal intersection and receives SPaT information once in DRSC range, four possibilities can occur (Kamalanathsharma & Rakha, 2014):

1) The time to the intersection (TTI) is less than the time to red (TTR), which is the time when the green light indication will change to red. Then, the optimal vehicle trajectory is to maintain its speed without acceleration or deceleration.
2) The time to the intersection (TTI) if the vehicle maintains its speed is larger than the time to red (TTR) but would make it through the intersection before the signal light switches to red if the vehicle accelerates to the maximum allowable speed limit. Then, the optimal speed profile is to accelerate to the speed required to pass the intersection during a green light indication.
3) The time to the intersection (TTI) is inadequate for the vehicle to pass during a green light indication even if it accelerates to the maximum allowable speed, and the time to green (TTG) of the next phase is significantly larger than the vehicle's time to the intersection (TTI). Therefore, the optimal solution is to completely stop at the intersection.
4) The time to the green of the next phase is not significantly larger than the time to intersection (TTI) so that the vehicle can alter its trajectory by decreasing its speed to a value so that it won’t have to come to a complete stop at the intersection and accelerate again to the desired speed, which is the optimal trajectory in this case.

The decision of the optimal vehicle trajectory is mainly based on the received information from the roadside unit (RU). In pre-timed or fixed traffic signals, the received information is deterministic and reliable for the trajectory optimization algorithm to take decisions. On the other hand, a deterministic estimate of the signal light's time to change is difficult to obtain for actuated or semi-actuated traffic signals because the control logic of actuated signals is continuously changing its SPaT information to accommodate traffic detected by the traffic sensors. As such, for the problem of trajectory optimization to be tractable, it is required to consider the uncertainty in the available estimations of the signal switching times. This problem can be thought of as a stochastic optimization problem, where there are multiple possibilities for signal switching times. For each possibility, there is a different trajectory that can minimize the objective function of reducing fuel consumption.

Some studies provided estimates of actuated signal timings switching from green to red and from red to green such as the work of (Eteifa et al., 2021) which utilized LSTM neural networks to predict reasonable estimates of signal switching times. The predicted switching times can be used together with their probabilities to formulate a two-stage stochastic program for the problem of vehicle trajectory optimization.

2.1. Impact of SPaT Uncertainty on Fuel Savings in Eco-Driving

Researchers have developed methods to address the uncertainty in SPaT predictions and achieve fuel savings despite uncertain information. However, there is a lack of information in the literature about how much the uncertainty in SPaT information affects achievable fuel savings. This makes it challenging to determine when a SPaT prediction is effective in reducing fuel consumption. Understanding the impact of SPaT prediction errors on fuel consumption can help assess the validity of SPaT predictions from an application standpoint and aid in choosing statistical or machine learning models. By examining prediction error distributions, better models can be selected, and their expected impact on fuel consumption savings with optimal eco-driving control can be determined. This study aims to investigate how the degree of uncertainty in SPaT predictions affects the achievable fuel savings when optimal vehicle control is implemented (Figure 1).

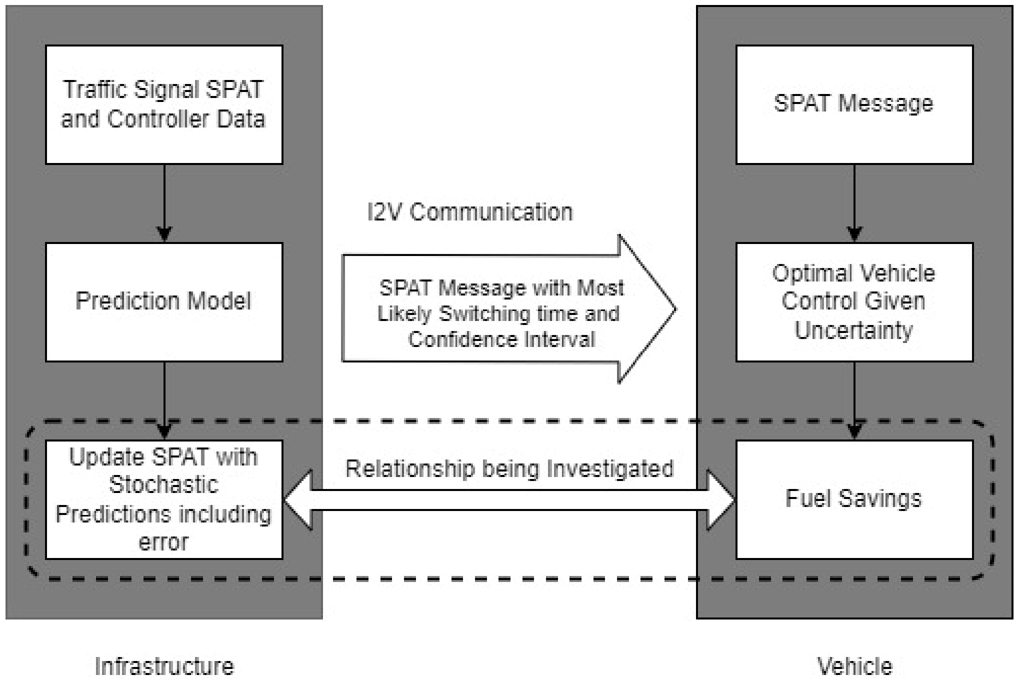

Figure 1: SPaT I2V Communication for Actuated Signals and Study Motivation

2.2. Objectives and Contribution

This study provides several contributions to the current literature and potential benefits for infrastructure operators and automotive original equipment manufacturers OEMs. It addresses the limitations found in previous research. The main contributions are:

- Extending the deterministic vehicle control algorithm proposed by Kamalanathsharma and Rakha (Kamalanathsharma & Rakha, 2014) to a stochastic algorithm that minimizes fuel consumption as an objective function while considering vehicle jerk constraints.
- Comparing the fuel consumption savings of the proposed system with actual field trajectories for uninformed drivers.
- Describing a framework for the secure implementation of GLOSA using stochastic predictions.
- Establishing benchmarks for practitioners who want to use statistical or machine learning models to forecast SPaT switching times based on both model bias and variance is needed to obtain the benefits of GLOSA.

Overall, this study enhances eco-driving control by creating a stochastic algorithm that explicitly minimizes fuel consumption, enabling the comparison of fuel consumption savings with real-world scenarios, and providing a framework for safe GLOSA implementation. Additionally, the benchmarks established can aid practitioners in selecting appropriate statistical or machine learning models to predict SPaT switching times accurately.

3. LITERATURE REVIEW

Vehicle trajectory optimization has been tackled extensively in the recent literature. Indeed, many researchers have addressed that problem in the latest two decades, such as (Wan et al., 2016) who developed a driver speed advisory system (SAS) that approached the problem of trajectory optimization as an optimal control problem formulation through which the optimal solution of the problem was obtained analytically. The optimization formulation minimized fuel consumption as a cost function. This study showed that the SAS has a significant effect on fuel minimization by optimizing vehicle trajectories near signalized intersections. Even with a low penetration level of this system, it has a harmonizing effect on other vehicles' motion in the vicinity and results in the overall decline of fuel consumption. However, this study considered all traffic signals as pre-timed or fixed signals and didn't apply the system to actuated signals (Wan et al., 2016). Kamalanathsharma et al. (Kamalanathsharma & Rakha, 2014) developed an application that optimizes vehicle trajectories by leveraging wireless communication in the DSRC range. The study utilized the received SPaT information and made predictions of future constraints that the vehicle would be affected by to optimize the vehicle trajectory. The study considered vehicle dynamic models using a polynomial fuel consumption model and explicitly formulated the objective function as a fuel minimization problem. Results of the developed application show that the saving of fuel consumption near a signalized signal can range between 5% and 30% depending on the speed of the approach. Though, like the previously reviewed study, the model only considered fixed-time signals and did not consider the actuated signals where the SPaT information is difficult to obtain. In addition, the study did not consider lateral movement in the model of vehicle dynamics (Kamalanathsharma & Rakha, 2014).

Most of the previous research has focused its work on the trajectory optimization of vehicles toward deterministic signal timings. The uncertainty and lack of exact information about signal phasing and timings (SPaT) make the deterministic approach of trajectory optimization inapplicable; however, as actuated signals are more widely deployed in the US. The literature shows that studies that consider the stochasticity of SPaT information received from actuated signals are overlooked. To the best of our knowledge, few researchers have considered actuated signals controllers. Mandava et al., 2009 developed an algorithm to generate a vehicle speed profile by giving speed instructions to the driver so that the probability of reaching the signalized intersection during the green time is maximized. The algorithm uses a stochastic simulation technique to generate samples of possible trajectories for a signalized corridor that has 10 signalized intersections. The developed algorithm resulted in energy savings of up to 14%. The study introduced some ways to deal with stochastic SPaT information. However, the objective function did not explicitly minimize fuel consumption or emissions. Instead, reaching the intersection during a green light was the primary goal. Another study by (Hao et al., 2015) developed an algorithm to estimate the maximum and minimum green time values for each phase of the actuated signal, and it recommended two possible trajectories based on the two estimates. The uncertainty decreases when the vehicle approaches closer to the intersection, yet the optimization algorithm solved the stochastic problem by two deterministic solutions corresponding to maximum or minimum values. However, this approach might not be as efficient as considering expected time to change.

Some studies approached the problem differently, such as the work of (Yao et al., 2020), which developed a joint optimization system that minimizes gasoline consumption of vehicles near actuated signals. In the upper level, the algorithm optimizes the actuated signal itself based on the predicted vehicle arrival which is calculated in the lower level. The vehicle trajectory is optimized in the lower level by a model predictive control. This study reported a decrease in fuel consumption and emissions by 22.36% and 18.61%, respectively (Yao et al., 2020). However, the runtime is significantly long as it uses a cyclic iteration method, which takes a long time to find the optimal trajectory, and thus, this algorithm is not practical for real-time implementation. In addition, the algorithm only considers the longitudinal vehicle movement and disregards the lateral movement. The algorithm modifies the actuated signal to accommodate the vehicle approaching the signal. If we envision that all vehicles at all approaches are equipped with the same algorithm, that will cause a contradiction between the optimization algorithm for each single vehicle trying to optimize the signal to accommodate its own movement, which makes this algorithm impractical.

Some researchers have recently sought to address the gap in research concerning the uncertainty of signal switching times. One such example is (Mousa et al., 2019), who developed a framework for trajectory optimization at semi-actuated signals in which only the minor approaches at an intersection have variable signal timings. The methodology was based on installing vehicle detectors 300m upstream of the stop line of the minor approaches. The algorithm resulted in 23.2% estimated fuel savings (Mousa et al., 2019). However, the framework has some drawbacks, such as the higher deployment costs generated by the installation of additional detectors. It also ignored turning movements and considered only through movements. Furthermore, the algorithm is not optimal to run in real-time situations as the optimal trajectory is found by a brute-force algorithm, which calculates all possible trajectories and selects the best one for implementation. This process is time consuming. On the other hand, as stated by the authors, the algorithm can be applied in real-time after running several simulations and storing enough data that can cover all possible scenarios that can occur.

In the next sections, the mathematical formulation of the problem introduced in part I is presented and discussed. Two suggested formulations in this report are then presented. First, the suggested stochastic problem formulation is presented. Then, decision variables, constraints, data variables, and the underlying models that are used in the formulation are demonstrated. Finally, a minor variant of the formulation is presented.

4. RESEARCH METHODOLOGY AND DATA PREPARATION

This research report presents an extension of the deterministic vehicle control algorithm proposed by Kamalanathsharma and Rakha (Kamalanathsharma & Rakha, 2014) to deal with actuated signal controllers. The problem of optimizing vehicle trajectory is formulated as a robust optimal control problem that considers uncertain signal switching times received through I2V communication. The system minimizes vehicle fuel consumption as an objective function and uses Dynamic Programming and A* algorithms to numerically solve the problem. A risk assessment procedure is implemented to ensure the algorithm's robustness towards uncertain signal information and prevent red light violations. The system's performance is evaluated by comparing it to the case of an uninformed driver approaching a traffic signal without prior information. The research methodology consists of defining the eco-driving problem, developing a holistic eco-driving system, evaluating the system's performance under different scenarios and levels of bias and variance, and identifying the effect of the vehicle's initial speed on the overall system performance.

This study provides several contributions to the existing body of knowledge and potential benefits to infrastructure operators and automotive equipment manufacturers:

- The proposed system addresses the drawbacks found in the literature and extends the deterministic vehicle control algorithm to a stochastic algorithm.
- The study presents a framework for the safe application of the proposed system given a stochastic prediction.
- The study identifies benchmarks for practitioners attempting to use statistical or machine learning models to predict SPaT most likely switching times.

Overall, the study aims to improve eco-driving control decisions, achieve fuel savings despite uncertain information, and enhance the safety and efficiency of transportation systems.

The eco-driving approach simulation results are compared to the case of an uninformed driver approaching a traffic signal without prior information about the switching time as a baseline. The uninformed driver data was retrieved from a field experiment conducted at the Virginia smart road test facility at the Virginia Tech Transportation Institute (VTTI) (Almannaa et al., 2019).

This study adopts a four-step research methodology described as follows:

- The first step involves defining the eco-driving problem and analyzing different scenarios.

- The second step is the development of a holistic eco-driving system that considers stochasticity and changing information while meeting vehicle and traffic signal constraints.
- The third step evaluates the system's performance under different pre-defined scenarios and various levels of bias and variance in predictions.
- The fourth step identifies how the vehicle's initial speed affects the overall system performance.

4.1. Analysis Scenarios

4.1.1 Base Scenario

In this study, the base scenario is defined as shown in Figure 2, which involves a single-lane approach with no other traffic and a free-flow speed of 40 mph (64.4 kph), which is typical for urban arterials. The vehicle starts at the position X_0, which is 250 meters upstream of the stop bar of an actuated traffic signal controller located at a position on X_M. The vehicle should accelerate to the desired speed by the position X_N, which is 180 meters downstream of the stop bar. The vertical grade is set at 3% either uphill or downhill. This setup is consistent with the field experiment used as a baseline for the study.

The assumption is that the vehicle is within the communication range of an actuated traffic signal controller and that the controller is transmitting SPaT (Signal Phase and Timing) information to the vehicle in real time. However, because actuated signals are stochastic, the SPaT information will include the most likely signal switching times.

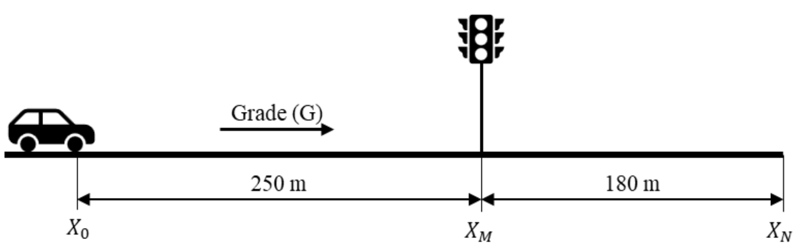

Figure 2: Problem Setup

4.1.2 Additional Analysis Scenarios

The study utilizes the same problem setup for various scenarios, which are outlined in TABLE 1.

- Scenario I involves an uninformed driver who approaches a red traffic signal and is unaware of the switching time. The switching time can take on values of 10, 15, 20, or 25 seconds. Vehicle trajectory data for this scenario is obtained from a previous field experiment conducted at the Smart road.

- Scenario II involves providing the vehicle with signal switching time information to optimize the trajectory. In Scenario II, the vehicle is provided with the exact switching time for a fixed time signal, and a deterministic vehicle trajectory optimization algorithm is used to plan the acceleration/deceleration policy.

- Scenario III is similar to Scenario II, in which an actuated traffic signal controller is simulated. However, the vehicle is provided with stochastic switching times that are sampled from normal distributions at each analysis time step. The mean of the distribution represents the true switching time plus an error bias value, while the standard deviation (STD) reflects the variability in switching time provided by prediction algorithms. The study tests different values of the bias and standard deviation to account for the quality levels of switching time prediction.

This uncertainty in the stochastic nature of switching time prediction is inherent in actuated traffic signals. The study aims to evaluate the performance of the vehicle trajectory optimization algorithm under various scenarios and levels of uncertainty. The simulation results are expected to provide insight into the effectiveness of the algorithm in minimizing energy consumption while adhering to the constraints imposed by the vehicle and the traffic signal.

TABLE 1 Analysis Scenarios

Scenario	Description	Directions	Initial Speed (mph)	Switching Times (TTG) (sec)
I (Baseline)	Uninformed Driver			
II	Deterministic SPaT Information	1 (Downhill) or 2 (Uphill)	40	10, 15, 20, 25
III	Stochastic real-time SPaT Information			

To create an effective tool for planning trajectories for green light optimal speed advisory (GLOSA), the issue is approached as a problem of stochastic optimal control. The system assumes that the exact switching time of the traffic signal is unknown, and instead uses the most probable or probabilistic distribution switching time to plan the vehicle's trajectory. This means that the system operates under the assumption that the signal information the vehicle receives in real-time is uncertain as it approaches the intersection at each period of time Δt. By incorporating this uncertainty into the system, the trajectory planning tool can adapt to changing traffic conditions in terms of the changing of traffic signal switching time and optimize the vehicle's speed to ensure it can pass through the intersection with the green light. This approach is crucial for developing a tool that can handle the stochastic nature of traffic signals and provide accurate recommendations to the driver, allowing them to save time, reduce fuel consumption, and minimize emissions. The stochastic optimal control problem formulation considers the uncertainty in the switching times, allowing for the development of a more robust and adaptable trajectory planning tool that can operate in various traffic conditions. Overall, the stochastic optimal control approach ensures that the tool is effective in optimizing the vehicle's speed while accounting for the uncertainty in the signal information, improving traffic flow, and reducing environmental impact.

5. MATHEMATICAL FORMULATION AND UNDERLYING SYSTEMS

5.1. Mathematical Formulation

The problem can be described as follows: We define a discrete time variable, denoted by t, which takes values from the set $\{t_0, \dots, t_f\}$. Here, t_0 is the timestamp when a vehicle enters the system, and t_f is the timestamp when it reaches its destination. The time interval between two consecutive timestamps is denoted by Δt, which is assumed to be 0.1 seconds in this specific context.

The purpose of Equation (1) is to minimize the total expected fuel consumption by considering the speed control policy, which is determined by the acceleration or deceleration level a_t at each timestamp t. The objective function sums up the expected values of fuel consumption, where FC_t represents the instantaneous fuel consumption at time t.

$$\text{Min} \sum_{t=t_0}^{t_f} \mathbb{E}\big(FC_t(a_t)\big) \cdot \Delta t \qquad (1)$$

Further, let x_t be a discretized distance variable at time t that belongs to the space $\mathbb{X} \subset \mathbb{R}^n$, and X_t is the vehicle position at time t (2). v_t is the speed variable at time t that depends the on acceleration/deceleration level a_t, which is governed by either the throttle input $f_{b,t}$ or the braking deceleration level.

$$X_t = \sum_{t_0}^{t} x_t = \sum_{t_0}^{t} v_t \cdot \Delta t \qquad (2)$$

The admissible speed policy space is constrained by the vehicle dynamics, acceleration, jerk, and other system control constraints that ensure safety and comfort.

The problem constraints are illustrated as follows: Equation (3) shows the upstream distance constraint, where the vehicle traveled upstream distance is equal to the traveled distance from the initial position X_0 at time t_0 until the time when the signal switches to green at position X_S. Knowing that the switching time is uncertain in this case, the expected value of the switching time t_s is used. The traveled upstream distance is upper bounded by the position of the stop bar X_M.

Note that using the expected value of the switching time denotes a risk-neutral attitude, where the upstream optimal trajectory is planned regardless of the risk of running a red light. However, this risk is eliminated by introducing a risk assessment procedure, where a critical stopping distance $d_{cr,t}$ is calculated at every time step t. When the remaining distance to the stop bar is less than or equal to $d_{cr,t}$, the vehicle policy is set to decelerate at the maximum allowable rate α (4). This maximum deceleration rate is set to $-6.0\ m/s^2$, which is considered a comfortable level of deceleration to stop. This is similar to an approach described in the literature that defined a last-resort parabola for the minimum stopping distance [26].

$$X_s = \sum_{t=t_0}^{\mathbb{E}(t_s)} v_t \cdot \Delta t \leq X_M \tag{3}$$

$$v_{t+\Delta t} = v_t + \alpha \cdot \Delta t \quad \forall\ t \leq \mathbb{E}(t_s), \forall\ X_t\ where\ X_M - X_t \leq d_{cr,t} \tag{4}$$

Similarly, the downstream speed policy is constrained to cover the downstream distance in addition to the remaining distance to the stop bar in the case when the signal switches before the vehicle reaches the position X_M (5). Note that this constraint is applied after the true switching time t_s is revealed.

$$\sum_{t=t_s}^{t_f} v_t \cdot \Delta t = X_N + (X_M - X_s) \tag{5}$$

The speed policy is further constrained by the kinematic equation (6), where a_t is the vehicle acceleration/deceleration level. Equation (7) shows the speed limit constraint. Finally, the acceleration policy is constrained by the maximum jerk limitation of $1.3\frac{m}{s^3}$ to ensure the passengers' comfort (8), where $f_{b,t}$ is the throttle input [27].

$$v_{t+\Delta t} = v_t + a_t \cdot \Delta t \quad \forall t \in \{t_0, \dots, t_f\} \tag{6}$$

$$v_t \leq v_{lim} \quad \forall t \in \{t_0, \dots, t_f\} \tag{7}$$

$$a_{t+\Delta t} \leq a_t + 1.3 \cdot \Delta t \qquad \forall t \in [t_0, \dots, t_f],\ \forall\ f_{b,t} > 0 \tag{8}$$

5.2. Underlying Systems

In order to determine the optimal trajectory for a vehicle, spatiotemporal variables are defined using vehicle dynamics models which take into account the current state variables and the forces acting on the vehicle such as tractive, aerodynamic, rolling, and grade resistance forces. The solution space is discretized in both time and space, and at each time step Δt, vehicle dynamics, and fuel consumption models are used to evaluate the optimal vehicle trajectory.

To model the vehicle's acceleration and deceleration behavior based on throttle and braking inputs, it is necessary to take into consideration all the forces and constraints on the vehicle. For the purposes of this research, a dynamic model for light-duty vehicles on varied terrain was used (Hesham Rakha et al., 2004), which computes the vehicle acceleration, tractive, and resistance forces as follows:

- The acceleration of the vehicle is determined by dividing the difference between the tractive force and the resisting force by the mass of the vehicle. This can be expressed mathematically as Equation 8, where F_t and R_t represent the tractive and resistance forces at time t.

$$a_t = \frac{F_t - R_t}{m} \tag{9}$$

- The force that the engine applies to the vehicle, known as the tractive force, is calculated. This force is limited by the maximum tractive force between the tires of the vehicle and the surface of the pavement (Equation 9). The equation includes several variables, such as the throttle input (f_b) which ranges from 0 to 1, the driveline efficiency (η_d), and a gear shift impact factor (β) which is set to 1.0 for light-duty vehicles (Hesham Rakha et al., 2004). Additionally, the equation includes the vehicle power at time t ($P_t$the vehicle speed at time t (v_t), the vehicle mass on the tractive axle (M_{ta}) in kilograms, the road friction coefficient (μ), and the gravitational acceleration (g) measured in meters per second squared.

$$F_t = \min\left[3600 f_{b,t}\eta_d \beta \frac{P(t)}{v(t)}, M_{ta} g \mu\right] \tag{10}$$

- The force that opposes the motion of the vehicle is the sum of three components, namely rolling, aerodynamic, and grade resistance forces. The formula used to calculate the resistance force is given as Equation 10, where ρ represents the density of air, C_d and C_h denote the drag coefficient and altitude correction factor of the vehicle, respectively. A_f is the frontal area of the vehicle, and c_{r0}, c_{r1} and c_{r2} are the constants for rolling resistance.

$$R(t) = \frac{\rho}{25.91} C_d C_h A_f v^2(t) + mg\frac{c_{r0}}{1000}(c_{r1}v(t) + c_{r2}) + mgG(t) \tag{11}$$

5.2.1 Fuel Consumption Model

To ensure that fuel consumption rates are accurately estimated and consistent with actual in-field measurements, the research team employed the Virginia Tech Comprehensive Power-Based Fuel Consumption model (VT-CPFM-1) (Rakha et al., 2011). This model has a number of advantages over existing fuel consumption models found in the literature, including the elimination of the bang-bang control behavior and the ability to be easily calibrated using publicly available vehicle data. Furthermore, it is known for its simplicity and accuracy in calculating instantaneous fuel consumption from instantaneous vehicle power. The formulation of the VT-CPFM-1 model used in this study is presented in Equation 11 below. More information on this model can be found in the literature (Rakha et al., 2011).

The VT-CPFM-1 model is able to accurately estimate fuel consumption rates because it takes into account a number of factors that contribute to fuel consumption, such as engine power, speed, and vehicle weight. By considering these factors, the model is able to provide a more realistic estimate of the amount of fuel consumed by the vehicle in a given time period. In addition, the model is able to account for the fact that fuel consumption rates can vary depending on the driving conditions, such as traffic congestion or changes in speed.

$$FC_t = \begin{cases} \alpha_0 + \alpha_1 P_t + \alpha_2 {P_t}^2 & \forall P_t > 0 \\ \alpha_0 & \forall P_t \leq 0 \end{cases} \tag{12}$$

Where α_0, α_1 and α_2 are the model constant calibrated for the specific vehicle in use. P_t is the instantaneous vehicle power calculated (13).

$$P_t = \left(\frac{R_t + 1.04\, m a_t}{3600 \eta_d}\right) v_t \tag{13}$$

In this context, the variables a_t and v_t represent the current acceleration and speed of the vehicle, respectively, and m refers to the vehicle's mass. The driveline efficiency is denoted by η_d. The study utilized a 2014 Cadillac SRX for the field test, and therefore the calibrated parameters for this vehicle were utilized to ensure comparability between the simulated results and the experimental results obtained in the field. The vehicle parameters, which can be found in Table 2, were used as a baseline for the calculations.

TABLE 2 Cadillac SRX 2014 Parameters

Parameter	Value	Parameter	Value
α_0	7.89E-04	A_f	3.33
α_1	-5.77E-19	η_d	0.92
α_2	2.27E-06	% Mass on tractive axle	0.54
c_{r0}	1.75	m	2388
c_{r1}	0.0328	P_{max}	229.7
c_{r2}	4.55	C_h	0.95
c_d	0.39	ρ	1.2256

5.2.2 Vehicle Dynamics Model

The utilization of vehicle dynamics models involves defining spatiotemporal variables based on current state variables and acting forces on the vehicle, such as tractive, aerodynamic, rolling, and grade resistance forces. The discretization of the solution space necessitates the use of vehicle dynamics and fuel consumption models to determine the optimal vehicle trajectory at each time interval Δt. To accurately model the vehicle's acceleration and deceleration, accounting for all acting forces and constraints, it is necessary to consider the throttle level or braking inputs. This research aims to optimize the trajectory of light-duty vehicles and employs a dynamic model for light-duty vehicles on varied terrain (Hesham Rakha et al., 2004). This model computes the vehicle acceleration, tractive, and resistance forces as described below.

- To calculate the vehicle's acceleration, the net force acting on the vehicle (which is the tractive force minus the resisting force) is divided by the vehicle mass m, as shown in Equation 9). The tractive force at time t is represented by $F(t)$, and the resistance force at time t is represented by $R(t)$.

$$a(t) = \frac{F(t) - R(t)}{m} \tag{14}$$

- The vehicle's tractive force is determined by the force generated by the engine, which has an upper limit of the maximum tractive force between the vehicle tires and the pavement, as illustrated in Equation 10. The calculation takes into account various factors, including the throttle input f_b (ranging from 0 to 1), driveline efficiency (represented by η_d), and the gear shift impact (represented by β, which is set to 1.0 for light-duty vehicles) (Hesham Rakha et al., 2004). Other variables included in the equation are the vehicle power (represented by $P(t)$), vehicle speed (represented by $v(t)$), vehicle mass on the tractive axle (represented by M_{ta} in kilograms), road friction or adhesion coefficient (represented by μ), and gravitational acceleration (represented by g in m/s^2).

$$F(t) = \min\left[3600 f_b \eta_d \beta \frac{P(t)}{v(t)}, M_{ta} g \mu\right] \tag{15}$$

- The total resistance force that acts on the vehicle is calculated by adding up the rolling, aerodynamic, and grade resistance forces, as shown in Equation 11. The equation includes several variables, such as air density (represented by ρ), the vehicle's drag coefficient (represented by C_d), and the altitude correction factor (represented by C_h). Other variables include the vehicle's frontal area (represented by A_f) and the rolling resistance constants (represented byc_{r0}, c_{r1} and c_{r2}).

$$R(t) = \frac{\rho}{25.91} C_d C_h A_f v^2(t) + mg \frac{c_{r0}}{1000} (c_{r1} v(t) + c_{r2}) + mgG(t) \quad (16)$$

5.3. Solution Approach

5.3.1 Stochastic Dynamic Programming

Due to the non-linear stochastic nature of the optimization problem at hand, determining the optimal speed policy can be computationally intensive. To make this algorithm more practical for real-world use, we need to decrease its complexity and find a heuristic solution that is sufficiently close to the optimal one in a reasonable amount of time. One way to accomplish this is by using dynamic programming (DP), which is a powerful method for solving stochastic optimization problems that works by discretizing time and utilizing Bellman's principle of optimality. DP is well-regarded for significantly reducing computation complexity. (Gianluca Fusai, 2008)

In our problem, we have only one control variable ($m = 1$), which can be either the throttle input or deceleration level. The problem is solved within a control time horizon of length $T = (t_f - t_0)/\Delta t$, which is equal to the time difference between the starting time t_0 of the simulation and the final time t_f when the vehicle reaches its destination at the position X_N, divided by the time step Δt. Therefore, the solution space is $\mathbb{R}^{m\times T}$. Dynamic programming provides a powerful tool to break down the problem into a sequence of T subproblems, each defined in the space $\mathbb{R}^{m}$. This decomposition significantly reduces the computational complexity of the problem.

As depicted in Figure 3, the solution approach is described as follows:

1. In the beginning, when the vehicle reaches the upstream link, it is in a position S_0 and the traffic signal is red.
2. The system uses the current speed to calculate the critical distance d_{cr} required to decelerate at the maximum allowable deceleration level ($\alpha = -6\ m/s^2$). The system also checks the remaining distance to the stop line at the signalized intersection, D_i.
3. While the traffic signal is red, the system evaluates the risk of running the red light by comparing the remaining distance to the intersection with the critical stopping distance. There are two possibilities:

a. If the remaining distance is less than or equal to the critical distance, the risk of running the red light is high, and the system adopts a deceleration policy with the rate α.
b. If there is no risk of red-light violation, the system receives SPaT information, generates the next state policy for the vehicle using the A* algorithm, and repeats steps 1 through 3.

4. If the traffic signal turns green, the system generates the next downstream policy states using the A* algorithm until the vehicle reaches the destination at the position S_M.

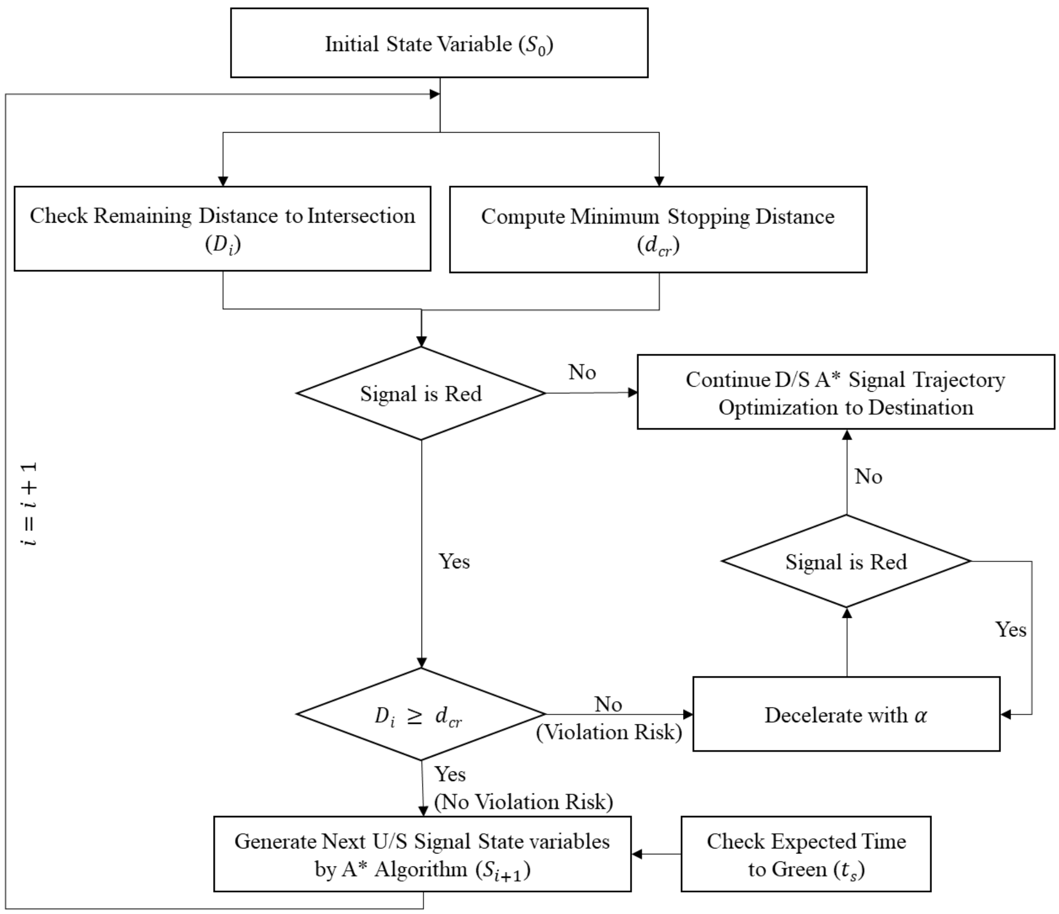

Figure 3 The Stochastic Setting Solution System

Safety Distance Buffer

Preventing the vehicle from running a red light is ensured through a risk assessment procedure which calculates the critical stopping distance $d_{cr,t}$ based on the current speed and a desired deceleration level as $v_t^2/2a_{des}$, where a_{des} is the desired deceleration level. When the vehicle's distance to the intersection is less than the critical distance, the vehicle will start decelerating to a stop as long as the signal indication is still red. This procedure is defined to prevent the vehicle from running a red light and to ensure that it decelerates at some acceptable level, which might happen due to the uncertainty in the switching time predictions. However, this procedure can also lead to unnecessary fuel losses when the vehicle starts decelerating, the signal switches to green, and then the vehicle accelerates back to the desired speed, losing some fuel.

To mitigate this issue, an enhancement is proposed by incorporating a real-time calculated safety time buffer (B). This buffer represents an additional delay imposed by the system to prevent the vehicle from reaching the critical distance to the intersection and activating the risk assessment procedure. As shown in Figure 4, the safety buffer (B) is calculated in real time using the expected time to green (TTG) and the critical distance $d_{cr,t}$, according to the analytical solution shown in (14). It is shown that the vehicle avoids reaching the critical distance to the intersection until the signal switches to green. It is also noted that in some cases, when the switching time is relatively long, the vehicle will have to stop anyway and idle at the intersection until the light turns green.

$$B = \frac{TTG}{\frac{d_u}{d_{cr,t}} - 1} \tag{17}$$

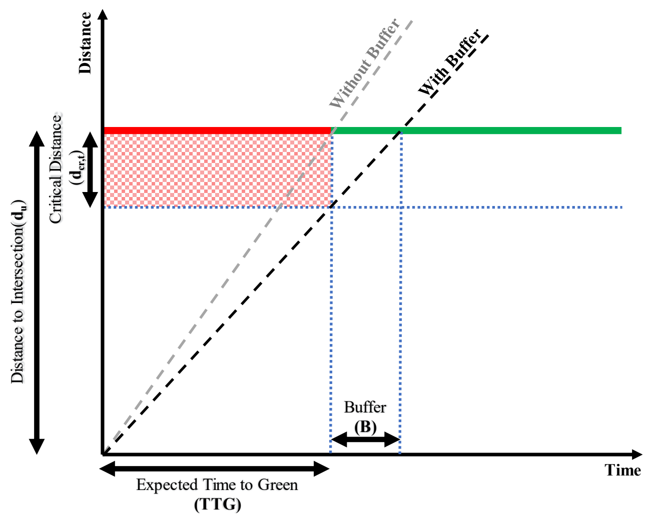

Figure 4: The effect of the safety time buffer (B) to prevent the vehicle from entering the critical distance to the intersection.

5.3.2 The A* Algorithm

The A* algorithm is a method for finding the shortest path by using a cost estimate that takes into account the expected cost of moving from the current state to the goal state. It has been applied in previous research to determine the trajectory of vehicles in situations where outcomes are certain (Kamalanathsharma & Rakha, 2014). In this study, the A* algorithm is employed to determine the next acceleration/deceleration policy with the least cost. This policy is then assumed to remain the same for the rest of the time horizon (Figure 5). The cost estimate heuristic in our problem is calculated over two sections: the upstream and downstream sections, which are described as follows:

1. The system performs an outer loop iteration for each upstream admissible policy in the upstream section. The algorithm assumes that each policy will remain the same until the vehicle reaches the position X_s. Policies that violate the expected red-light condition are considered infeasible. The system then computes the fuel consumption for each feasible policy i in the upstream section U_i.
2. For each feasible policy i, the system performs an inner loop iteration to generate admissible downstream policies based on the vehicle's state at position X_s. Similar to the upstream section, each downstream policy is assumed to remain the same until the vehicle reaches the destination at the position X_N. The system computes the fuel consumption for each upstream policy i and downstream policy j in the downstream section D_{ij}.
3. Using the heuristic cost estimates for both the upstream and downstream sections, the A* algorithm selects the upstream next-state policy with the minimum total fuel consumption of the two sections (U_i+D_{ij}) for each upstream policy i.
4. Once the vehicle reaches the downstream section, the system only iterates in the inner loop to select the policy with the minimum fuel consumption D_{ij}.

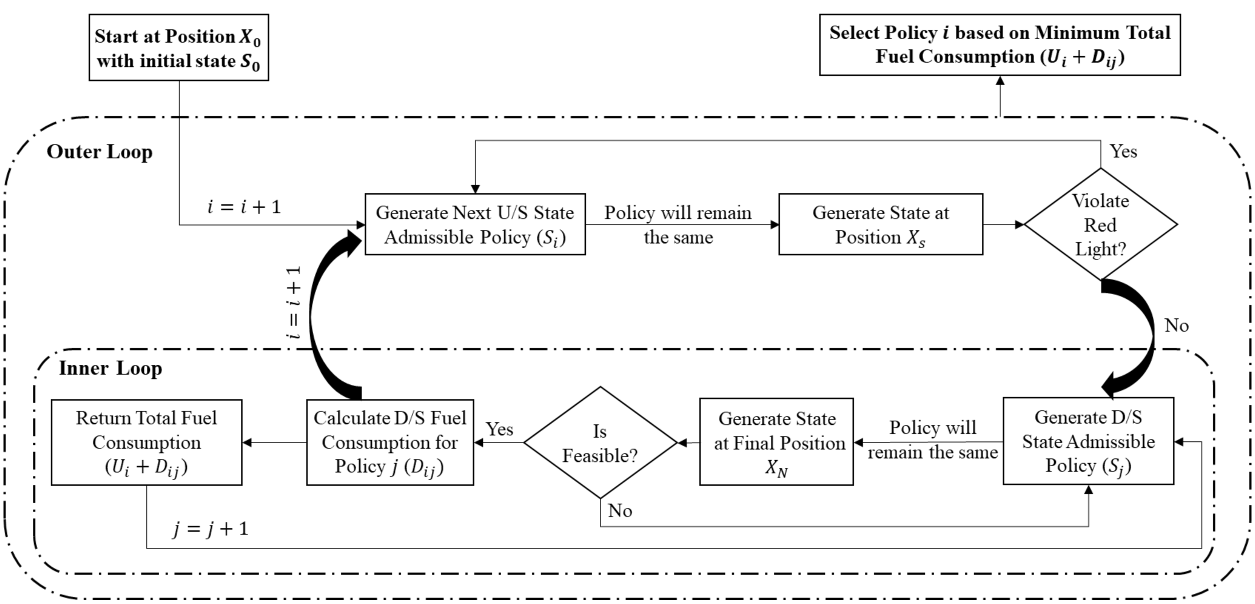

Figure 5: Logic of A-Star Algorithm to Find the Minimum Path

6. ANALYSIS RESULTS

6.1. Optimal Policy for Deterministic SPaT information

To solve our problem in specific scenarios listed in TABLE 1, the trajectory optimization system is used when the SPaT information is deterministic. Once the system receives the signal switching time from the controller, it computes the optimal trajectory that results in the best fuel economy. Figure 6 and Figure 7 demonstrate that when the time to green (TTG) is 10 seconds, the optimal policy is to maintain the current speed because the time to reach the intersection is greater than the time to green. To minimize fuel losses, the system uses the minimum throttle level required to overcome the resistance forces and maintain the vehicle's speed. When the TTG is 15, 20, or 25 seconds, the system recognizes that the vehicle needs to be delayed before reaching the stop line and applies the optimal deceleration levels to achieve the desired delay. For all cases downstream of the traffic signal, the system determines the optimal acceleration policy to reach the desired speed at the destination position.

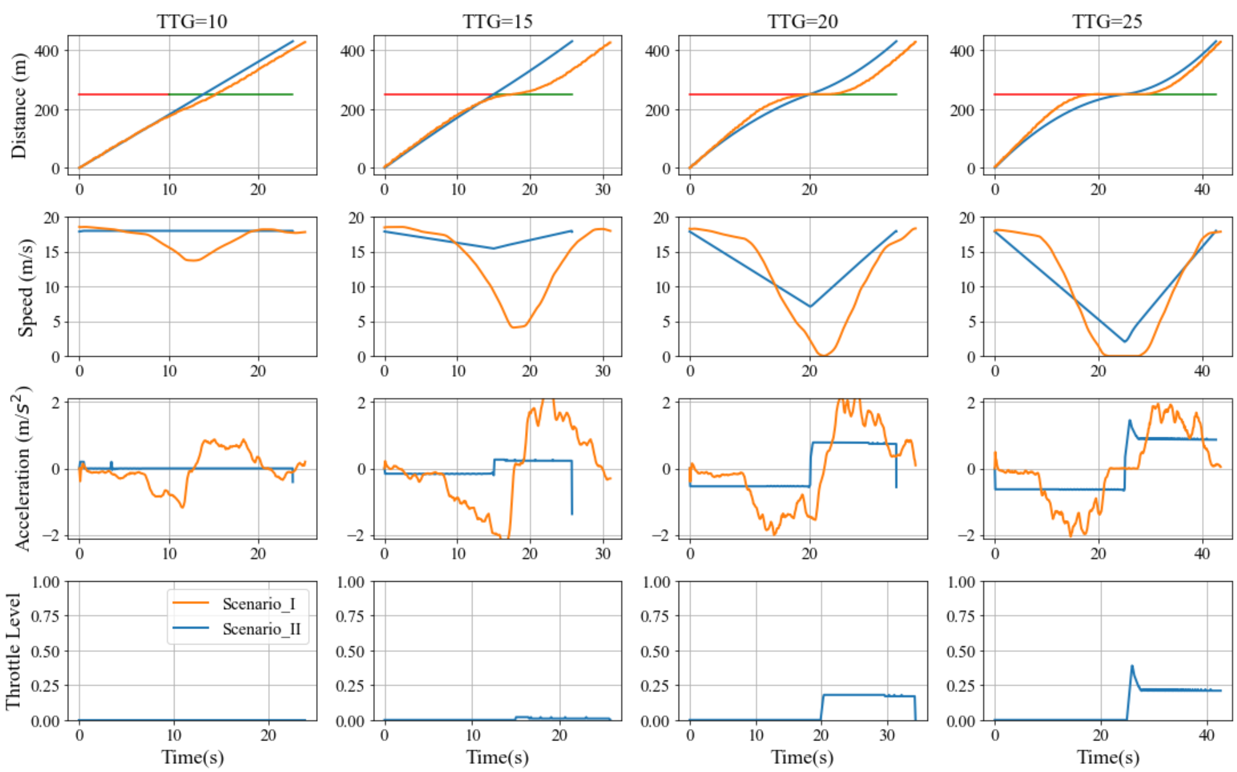

Figure 6: Comparison of Trajectories for Downhill Grade: Uninformed Driver (Scenario I) vs. Optimal Deterministic Trajectory (Scenario II)

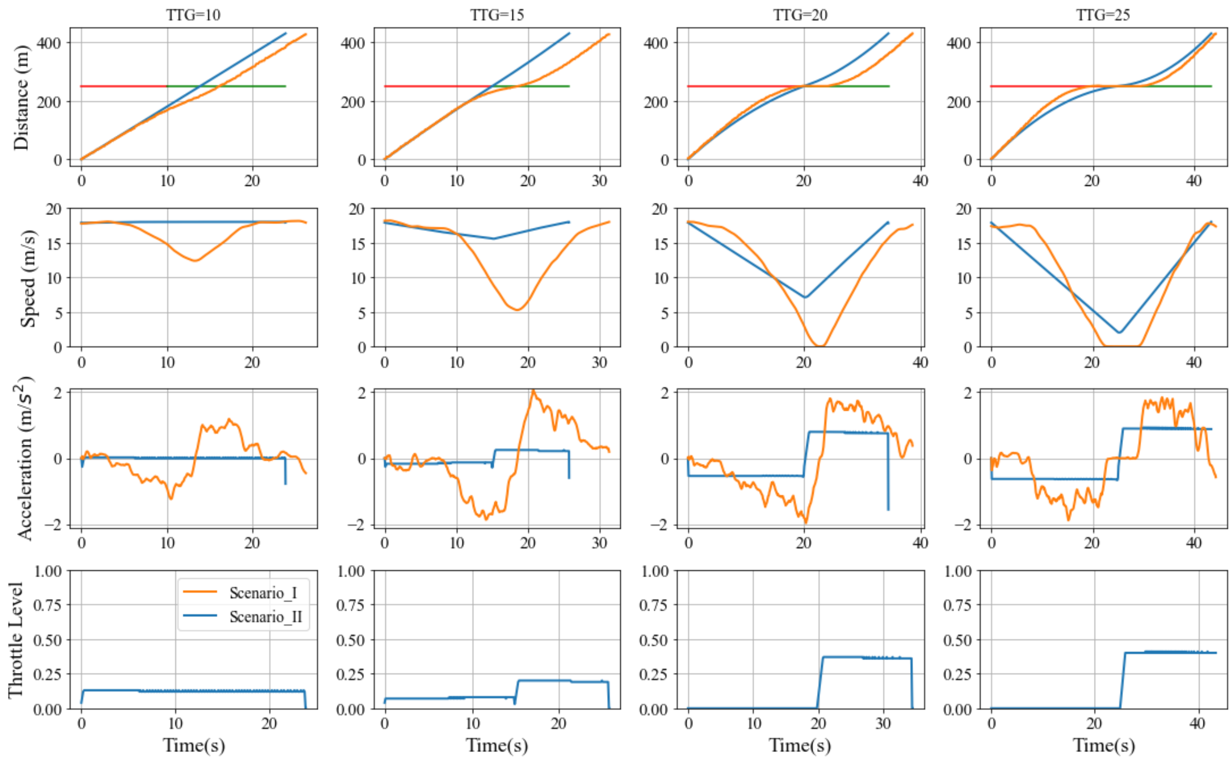

Figure 7: Comparison of Trajectories for Uphill Grade: Uninformed Driver (Scenario I) vs. Optimal Deterministic Trajectory (Scenario II)

6.2. Optimal Policy for Stochastic SPaT information

When the exact signal switching time is not provided, signal switching time predictions are used to plan the vehicle's trajectory. In simulations, these predictions mimic stochastic SPaT data received from a traffic signal controller. The prediction uncertainty is represented by a constrained normal distribution with two components: a bias value added to the mean and a standard deviation reflecting random errors in the SPaT prediction model (SD). Bias and SD values are sampled up to a maximum predefined value of 8 seconds, and a new random distribution is generated at each time interval based on the vehicle's position.

The system is used in various scenarios described in Table 1 with different probability distributions. At each time interval (Δt), the system receives a value sampled from the switching time probability distribution. Based on this information, the system generates a policy that maximizes fuel efficiency while considering the available information. This results in a variable policy behavior, as seen in Figure 8 and Figure 9. The system regulates the intensity of policy changes based on the jerk limit to ensure passenger comfort levels are not compromised.

If the exact signal switching time is unknown and the time-to-green (TTG) is 10 seconds, the system's ideal strategy is to keep moving at the current speed until it reaches the risk area. Once the vehicle enters the risk area, it decelerates until it comes to a stop if the signal is still red. The sudden drop in the acceleration profile in Figure 8 illustrates this risk assessment method. When the traffic signal turns green, the vehicle accelerates again to reach its destination at the desired speed.

Likewise, in cases where TTG is 15 and 20 seconds, the system creates the best deceleration plan while constantly monitoring the danger zone. If the danger zone is reached, the car activates the deceleration policy and stops as long as the signal is still red. If the light turns green during deceleration, the vehicle will modify its plan to accelerate and attain the desired speed at the destination position.

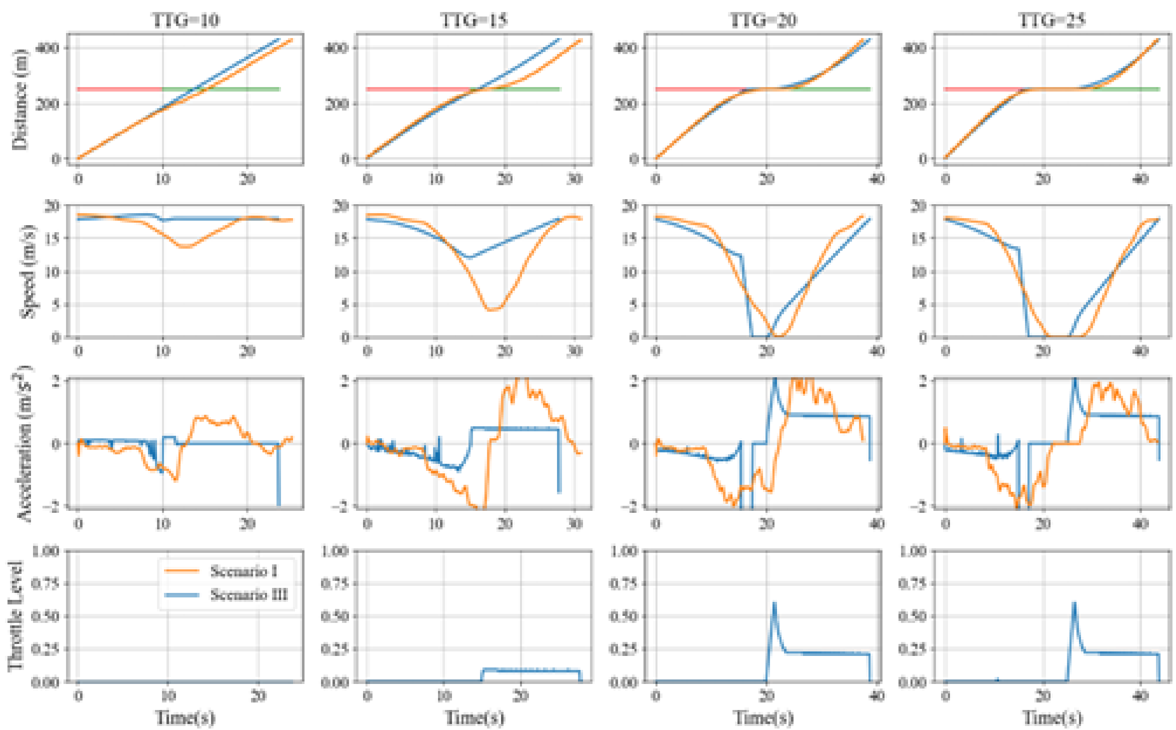

Figure 8: Comparison of Trajectories for Downhill Grade: Uninformed Driver (Scenario I) vs. Optimal Stochastic Trajectory (Scenario III)

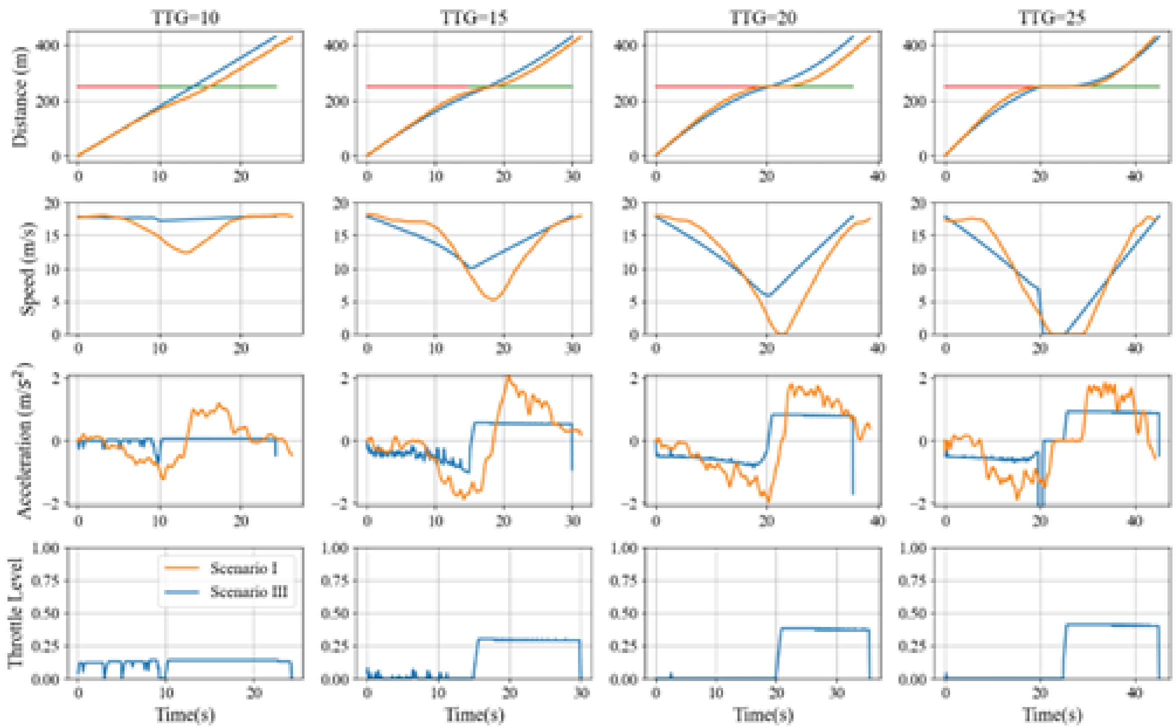

Figure 9 Comparison of Trajectories for Uphill Grade: Uninformed Driver (Scenario I) vs. Optimal Stochastic Trajectory (Scenario III)

6.3. Fuel Consumption Savings for Deterministic and Stochastic Settings

The study compared the fuel consumption of an informed driver using an optimization system to an uninformed driver in both deterministic and stochastic settings. The results showed that the optimization system significantly reduced fuel consumption in both cases. The stochastic SPaT information was used to mimic real-world traffic signal data, and the results were averaged over different levels of bias and standard deviations. The overall average fuel savings were 37% and 28% for deterministic and stochastic settings, respectively, as shown in Figure 10. The fuel savings were highest when the car was moving downhill with a TTG of 15 seconds, reaching up to 63% for deterministic and 42% for stochastic TTG. This is because the minimal throttle is needed to overcome resisting forces and make the vehicle cruise downhill.

The study found that TTG=15 is the most efficient setting because the optimal policy requires only a small deceleration to reach the stop bar by the time the signal turns green. The starting speed of the vehicle is 40 mph, and the average speed needed to reach the stop bar at the time the signal turns green is 37.3 mph. In a stochastic setting, the policy becomes more conservative, and more than needed deceleration is applied to account for the possibility that the SPaT information is not entirely accurate. This ensures that the vehicle stops in time, even if the prediction is incorrect. An uninformed driver would not have this information and would be more likely to stop, resulting in higher fuel consumption.

The study also demonstrated that the closer the time to green is to the time needed to reach the stop bar, the more sensitive the fuel consumption is to the accuracy of the prediction. Therefore, accurate SPaT information is critical for optimizing fuel consumption in stop-and-go traffic. The optimization system's effectiveness in reducing fuel consumption highlights the importance of incorporating intelligent transportation systems in modern vehicles to improve fuel efficiency and reduce emissions. These systems can provide valuable information to drivers and help them make informed decisions that result in lower fuel consumption and environmental impact.

The study investigated the effect of traffic signal switching times on fuel consumption in vehicles, considering both deterministic and stochastic scenarios. When the time to green (TTG) is 25 seconds, the vehicle cannot reach the stop bar without stopping, so fuel savings are achieved mainly by optimizing the acceleration profile in the downstream section. In this case, the fuel savings achieved by the optimization system with deterministic and stochastic SPaT information are similar, at 21% and 20% respectively, compared to an uninformed driver.

However, as the TTG decreases, the difference between the deterministic and stochastic scenarios becomes more pronounced, as optimizing the trajectory in the upstream section becomes increasingly important in reducing fuel consumption. The study found that optimizing the upstream trajectory can result in additional fuel savings of up to 22% for deterministic SPaT and 19% for stochastic SPaT. This emphasizes the significance of having accurate and reliable predictions for traffic signal switching times to enable efficient trajectory planning and fuel-saving optimization.

Overall, the study showed that the optimization system was able to achieve significant fuel savings compared to an uninformed driver, with average savings of 37% and 28% for the deterministic and stochastic scenarios, respectively. The maximum fuel savings were achieved in the downhill setting with TTG=15 seconds. The study also demonstrated that the closer the time to green is to the time needed to reach the stop bar at the current vehicle speed, the more sensitive fuel consumption is to the accuracy of the prediction.

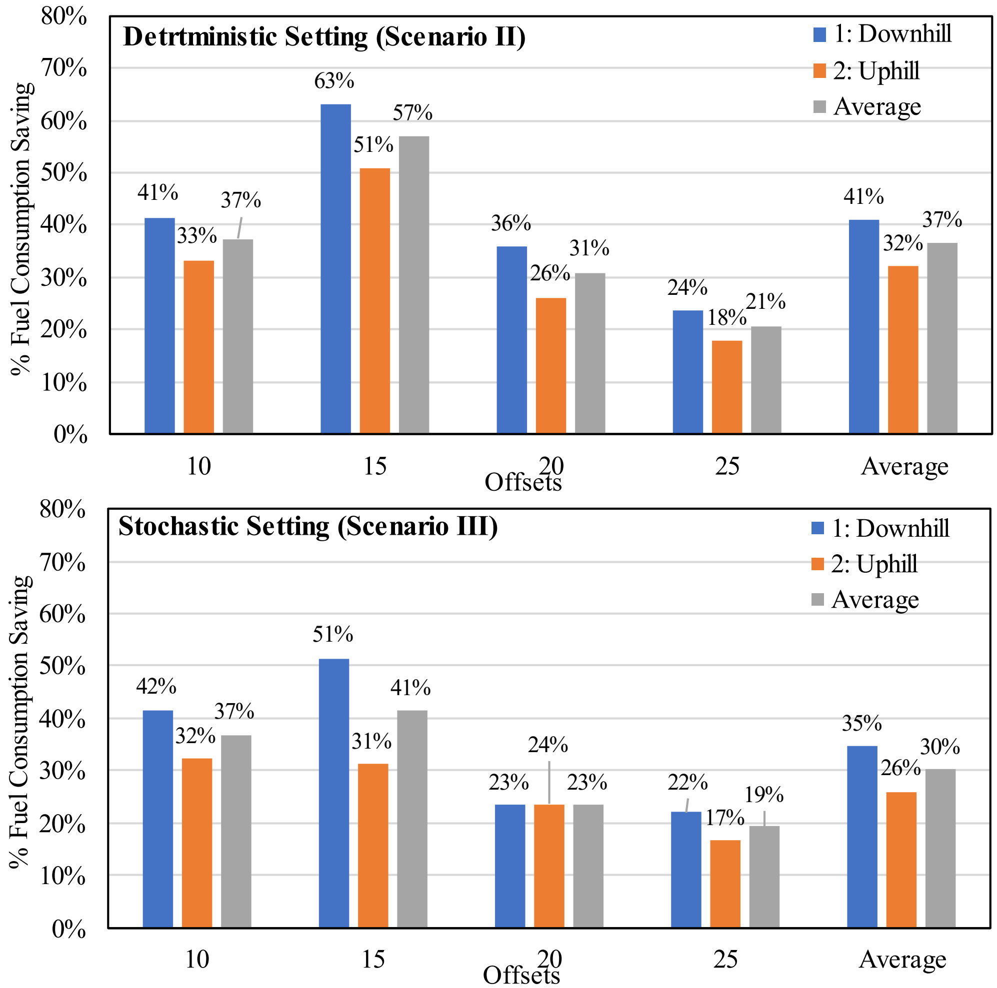

Figure 10: Fuel Consumption Savings of Scenario II and III over Scenario I

6.4. Effect of Bias and Variance on the Trajectory Planning Algorithm

The impact of the accuracy of traffic signal prediction on fuel consumption was analyzed by conducting a sensitivity analysis that varied the normal distribution parameters of error bias and standard deviation. A total of 320 runs were performed using different values of bias and SD (Figure 11). The results showed that the change in SD values had a significant effect on fuel consumption, whereas the effect of the bias value was not significant. Generally, additional uncertainty in the prediction led to fuel savings losses, except for the case where the time to green (TTG) was 15 seconds.

In the case of TTG=15s, the optimal fuel consumption levels decreased as the stochasticity of the prediction increased. This behavior can be explained by considering the vehicle's travel time from the initial position to the intersection, which equals the switching time, based on the initial speed of 40 mph. The vehicle approaches the stop line just as the signal is about to switch to green. However, the system activates a risk assessment procedure to ensure that no violations of traffic signal timings occur. Therefore, the vehicle begins to decelerate until the signal switches to green.

By introducing additional stochasticity, the vehicle can be delayed enough so that the signal will switch before the vehicle enters the risk zone and activates the stopping strategy. This is the only case when the more stochastic the information is, the better fuel economy can be achieved.

Figure 12 illustrates the relationship between fuel consumption and stochasticity in the case of TTG=15s. The graph shows a downward trend in fuel consumption as stochasticity increases, which is consistent with the explanation above. It demonstrates that the optimal fuel consumption policy for this scenario is a delicate balance between minimizing deceleration and ensuring that the vehicle does not violate traffic signal timings. Therefore, the stochasticity of the prediction plays a crucial role in achieving fuel savings in this particular scenario.

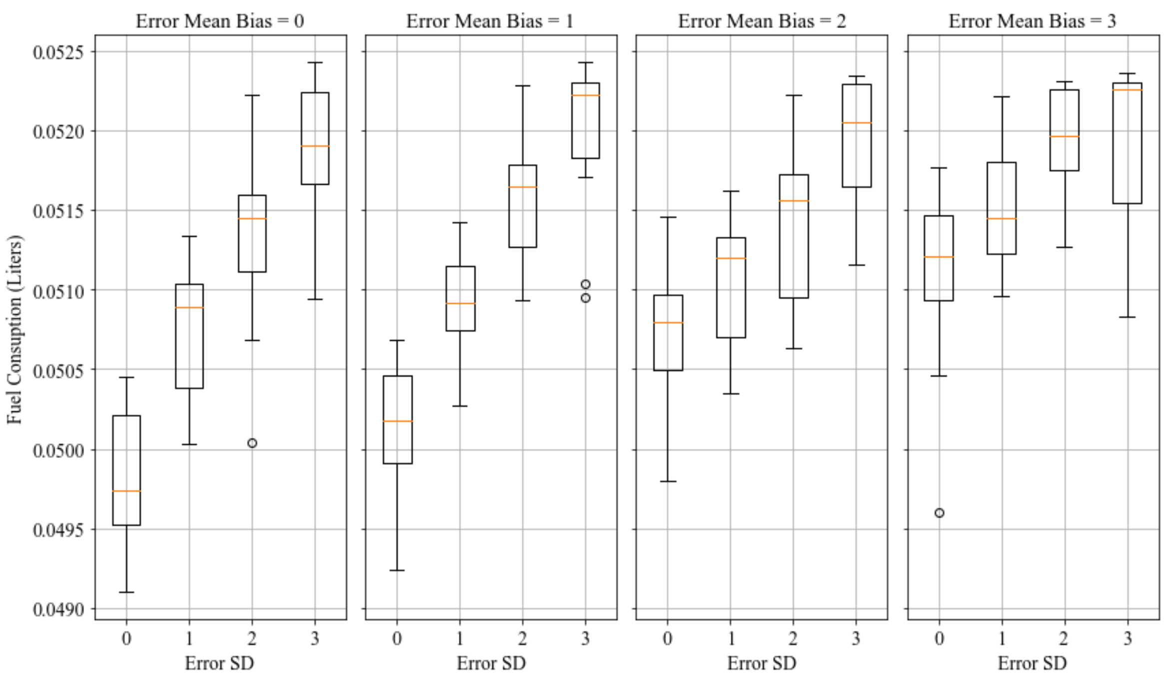

Figure 11: Fuel Consumption vs. Bias and SD Values for TTG=20s, Downhill Grade

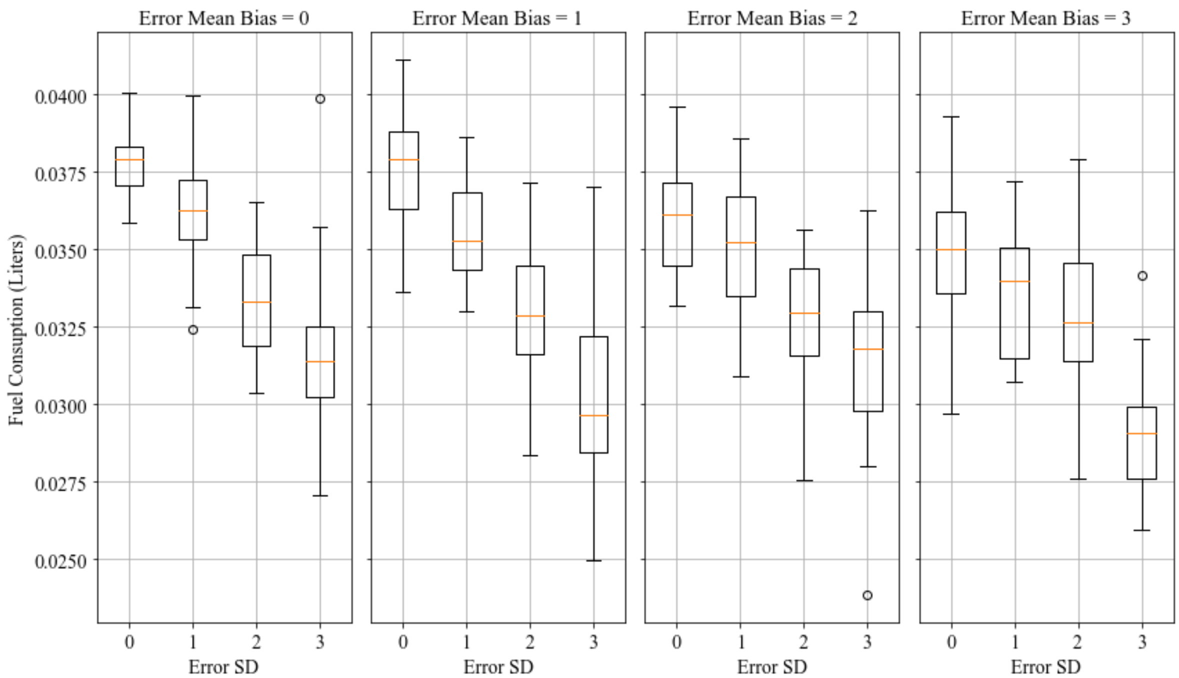

Figure 12: Fuel Consumption vs. Bias and SD Values for TTG=15s, Downhill Grade

6.4.1 The Effect of Varying Initial Speed

To test the response of the system to stochastic SPaT information, the initial vehicle speed was varied from 40 mph to 29 mph (Figure 13). The analysis showed that when the initial speed was lower, the system generated a nearly constant policy for TTG of 10 and 15 seconds. The policy was to provide minimum constant acceleration to reach the stop bar within the green and then reach the destination at maximum speed. This was because the optimal policy allowed the vehicle to arrive late enough to accommodate the switching time probability distribution for TTG of 10 and 15 seconds. For TTG of 20 and 25 seconds, a regularly fluctuating acceleration policy was generated to reduce the speed of the vehicle enough to reach the intersection before the signal turned green. However, a slow deceleration was followed by a sharp deceleration as the vehicle approached the safe stopping distance from the stop bar. This indicates that the prediction accuracy becomes more critical as the time to green gets closer to the time until reaching the intersection given the initial speed.

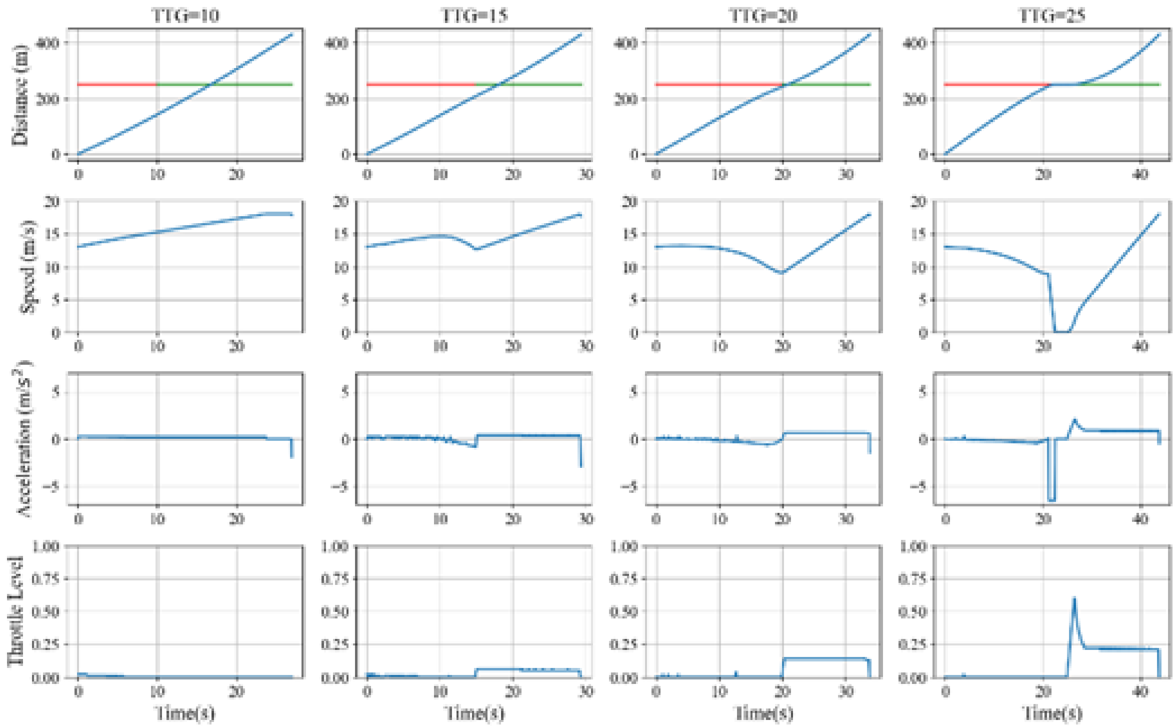

Figure 13: Optimal Trajectory Plots of Vehicle at Initial Speed of 13m/s (29mph).

6.5. Impact of Confidence in SPaT Information on the Fuel Consumption

This section discusses the importance of the level of confidence required in switching time prediction to reduce fuel consumption in a stochastic setting. The upper bound of fuel consumption savings is achieved in the deterministic setting, so this section aims to identify how the level of uncertainty impacts the savings. Figure 14 shows the relationship between the proportion of fuel saving and the parameters of the switching time probability distribution. The maximum fuel saving occurs when there is no uncertainty in switching time information, and the savings proportion decreases as the standard deviation and mean bias increase. This section concludes that fuel savings of more than 85% can be achieved in the stochastic setting when the standard deviation is less than 1.25 seconds, and the mean bias is less than 0.8 seconds. This means that a confidence level of 95% can be achieved with a switching time prediction error of up to ±3.3 seconds, providing insights into the required level of confidence in SPaT prediction to achieve significant fuel savings.

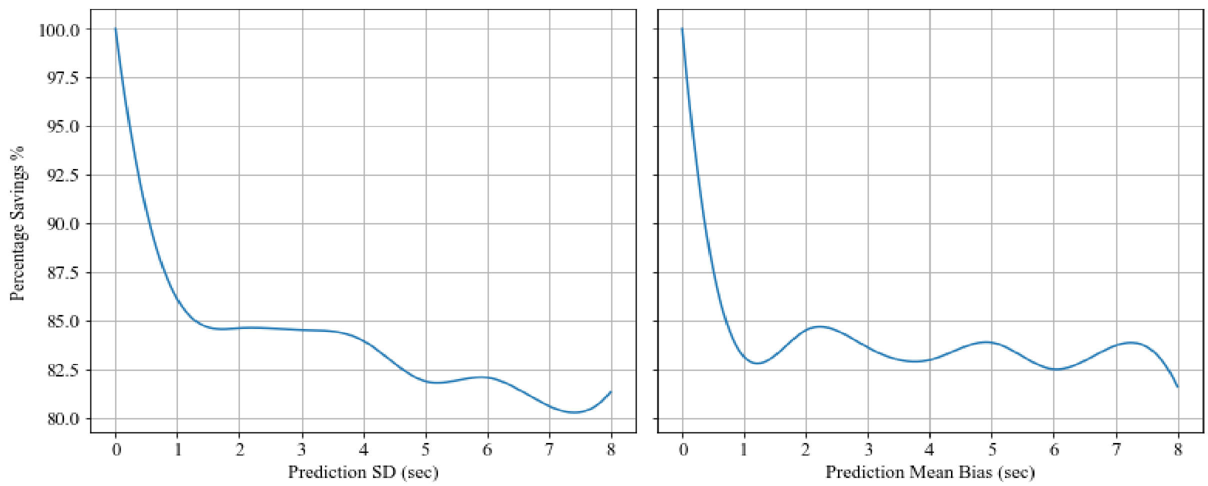

Figure 14 Relationship between Standard Deviation and Bias of Switching Time Probability Distribution and Fuel Consumption Savings in Stochastic Scenario II

7. CONCLUSIONS AND RECOMMENDATIONS

This report discusses the development of an optimal Green Light Optimal Speed Advisory (GLOSA) system designed to find the optimal trajectory for a vehicle approaching a traffic signal controller, taking into account fixed and actuated traffic signals where the exact signal switching time is unknown. The I2V provides a probability distribution for the switching time to the vehicle. The objective function is to minimize fuel consumption, solved through a Dynamic Programming (DP) procedure utilizing the A-Star algorithm to find the minimum-cost path. A risk assessment procedure is implemented to control the vehicle's acceleration and deceleration levels so the red light is not violated, and passengers' comfort is achieved by controlling the vehicle acceleration jerk to limit the disruption due to the error fluctuations in the expected switching time at each time step.

Simulation results show that significant fuel savings can be achieved, with an average of 37% and 28% for the deterministic and stochastic settings, respectively. Additionally, the system is resilient to the errors inherent to uncertain switching time predictions, as it is able to adjust the vehicle trajectory in real time according to the updated predicted probability distribution of the switching time. The system is also sensitive to prediction errors when the time to green is close to the time required for the vehicle to reach the intersection given its current speed.

The proposed system can achieve more than 85% of the possible savings achieved in the case of fixed time signals if the timing error is (± 3.3 seconds) at a 95% confidence level. It was shown that initial speed affects vehicle acceleration and deceleration fluctuations, which can be useful in planning trajectories in a corridor that has consecutive signal controllers.

Future research could explore the stochastic optimizer for different levels of market penetration within surrounding uninformed drivers when queues form upstream of the traffic signal, and for different levels of traffic congestion, as was done in the case of deterministic signal timings (Ala et al., 2016; Yang et al., 2016; Yang et al.).

8. REFERENCES

Ala, M. V., Yang, H., & Rakha, H. (2016). Modeling Evaluation of Eco–Cooperative Adaptive Cruise Control in Vicinity of Signalized Intersections. *Transportation Research Record: Journal of the Transportation Research Board*(2559), 108-119.

Almannaa, M. H., Chen, H., Rakha, H. A., Loulizi, A., & El-Shawarby, I. (2019). Field implementation and testing of an automated eco-cooperative adaptive cruise control system in the vicinity of signalized intersections. *Transportation Research Part D: Transport and Environment*, *67*, 244-262. https://doi.org/10.1016/j.trd.2018.11.019

David Schrank, Bill Eisele, Tim Lomax, & Bak, J. *Urban Mobility Scorecard*.

EPA. *Inventory of U.S. Greenhouse Gas Emissions and Sinks 1990-2019*. https://www.epa.gov/ghgemissions/inventory-us-greenhouse-gas-emissions-and-sinks

Eteifa, S., Rakha, H. A., & Eldardiry, H. (2021). Predicting Coordinated Actuated Traffic Signal Change Times using Long Short-Term Memory Neural Networks. *Transportation Research Record: Journal of the Transportation Research Board*, *2675*(9), 127-138. https://doi.org/10.1177/03611981211000748

Gaspard, M. E., & Vladimirsky, A. (2022). Optimal Driving Under Traffic Signal Uncertainty. *arXiv preprint arXiv:2201.04521*.

Gianluca Fusai, A. R. (2008). Dynamic Programming for Stochastic Optimization. In *Implementing Models in Quantitative Finance: Methods and Cases* (1 ed., pp. 69-82). Springer Berlin, Heidelberg. https://doi.org/https://doi.org/10.1007/978-3-540-49959-6

Hao, P., Wu, G., Boriboonsomsin, K., & Barth, M. J. (2015). *Preliminary evaluation of field testing on Eco-Approach and Departure (EAD) application for actuated signals* 2015 International Conference on Connected Vehicles and Expo (ICCVE),

Hesham Rakha, Matthew Snare, & Dion, F. (2004). Vehicle Dynamics Model for Estimating Maximum Light-Duty Vehicle Acceleration Levels. *Transportation Research Record: Journal of the Transportation Research Board*, *1883*, 40-49.

Kamalanathsharma, R. K., & Rakha, H. A. (2014). Leveraging Connected Vehicle Technology and Telematics to Enhance Vehicle Fuel Efficiency in the Vicinity of Signalized Intersections. *Journal of Intelligent Transportation Systems*, *20*(1), 33-44. https://doi.org/10.1080/15472450.2014.889916

Mahler, G., & Vahidi, A. (2014). An Optimal Velocity-Planning Scheme for Vehicle Energy Efficiency Through Probabilistic Prediction of Traffic-Signal Timing. *IEEE Transactions on Intelligent Transportation Systems*, *15*(6), 2516-2523. https://doi.org/10.1109/tits.2014.2319306

Mandava, S., Boriboonsomsin, K., & Barth, M. (2009, 4-7 Oct. 2009). Arterial velocity planning based on traffic signal information under light traffic conditions. 2009 12th International

IEEE Conference on Intelligent Transportation Systems,

Monthly Energy Review. (2021). https://www.eia.gov/totalenergy/data/monthly/pdf/mer.pdf

Mousa, S. R., Ishak, S., Mousa, R. M., & Codjoe, J. (2019). Developing an Eco-Driving Application for Semi-Actuated Signalized Intersections and Modeling the Market Penetration Rates of Eco-Driving. *Transportation Research Record: Journal of the Transportation Research Board*, *2673*(5), 466-477. https://doi.org/10.1177/0361198119839960

Mousa, S. R., Ishak, S., Mousa, R. M., Codjoe, J., & Elhenawy, M. (2020). Deep Reinforcement Learning Agent with Varying Actions Strategy for Solving the Eco-Approach and Departure Problem at Signalized Intersections. *Transportation Research Record: Journal of the Transportation Research Board*, *2674*(8), 119-131. https://doi.org/10.1177/0361198120931848

Rakha, H. A., Ahn, K., Moran, K., Saerens, B., & Bulck, E. V. d. (2011). Virginia Tech Comprehensive Power-Based Fuel Consumption Model: Model development and testing. *Transportation Research Part D: Transport and Environment*, *16*(7), 492-503. https://doi.org/10.1016/j.trd.2011.05.008

Sun, C., Guanetti, J., Borrelli, F., & Moura, S. J. (2020). Optimal Eco-Driving Control of Connected and Autonomous Vehicles Through Signalized Intersections. *IEEE Internet of Things Journal*, *7*(5), 3759-3773. https://doi.org/10.1109/jiot.2020.2968120

Wan, N., Vahidi, A., & Luckow, A. (2016). Optimal speed advisory for connected vehicles in arterial roads and the impact on mixed traffic. *Transportation Research Part C: Emerging Technologies*, *69*, 548-563. https://doi.org/10.1016/j.trc.2016.01.011

Wang, C., Zhao, X., Fu, R., & Li, Z. (2020). Research on the Comfort of Vehicle Passengers Considering the Vehicle Motion State and Passenger Physiological Characteristics: Improving the Passenger Comfort of Autonomous Vehicles. *Int J Environ Res Public Health*, *17*(18). https://doi.org/10.3390/ijerph17186821

Yang, H., Ala, M. V., & Rakha, H. A. (2016). Eco-Cooperative Adaptive Cruise Control at Signalized Intersections Considering Queue Effects. Transportation Research Board 95th Annual Meeting,

Yang, H., Almutairi, F., & Rakha, H. A. Eco-Driving at Signalized Intersections: A Multiple Signal Optimization Approach. *IEEE Transactions on Intelligent Transportation Systems*. https://doi.org/10.1109/TITS.2020.2978184

Yao, Z., Zhao, B., Yuan, T., Jiang, H., & Jiang, Y. (2020). Reducing gasoline consumption in mixed connected automated vehicles environment: A joint optimization framework for traffic signals and vehicle trajectory. *Journal of Cleaner Production*, *265*. https://doi.org/10.1016/j.jclepro.2020.121836